

NOTES ON DIGITAL SIGNAL PROCESSING

Practical Recipes for Design, Analysis, and Implementation

C. BRITTON RORABAUGH

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Contents

Preface	xi
About the Author	. xiii

PART I DSP Fundamentals

Note 1	Navigating the DSP Landscape	.1-1
Note 2	Overview of Sampling Techniques	. <mark>2-1</mark>
Note 3	Ideal Sampling	. <mark>3-1</mark>
Note 4	Practical Application of Ideal Sampling	. <mark>4-1</mark>
Note 5	Delta Functions and the Sampling Theorem	_. 5-1
Note 6	Natural Sampling	_. 6-1
Note 7	Instantaneous Sampling	. <mark>7-1</mark>
Note 8	Reconstructing Physical Signals	. <mark>8-1</mark>

PART II Fourier Analysis

Note 9	Overview of Fourier Analysis	9 -1
Note 10	Fourier Series	10-1
Note 11	Fourier Transform	11-1
Note 12	Discrete-Time Fourier Transform	12-1
Note 13	Discrete Fourier Transform	13-1
Note 14	Analyzing Signal Truncation	<mark>14-1</mark>
Note 15	Exploring DFT Leakage	15-1
Note 16	Exploring DFT Resolution	16-1

PART III Fast Fourier Transform Techniques

Note 17	FFT: Decimation-in-Time Algorithms
Note 18	FFT: Decimation-in-Frequency Algorithms
Note 19	FFT: Prime Factor Algorithm
Note 20	Fast Convolution Using the FFT

PART IV Window Techniques

Note 21	Using Window Functions: Some Fundamental Concepts
Note 22	Assessing Window Functions: Sinusoidal Analysis Techniques
Note 23	Window Characteristics
Note 24	Window Choices
Note 25	Kaiser Windows

PART V Classical Spectrum Analysis

Note 26	Unmodified Periodogram
Note 27	Exploring Periodogram Performance: Sinusoids in Additive White Gaussian Noise
Note 28	Exploring Periodogram Performance: Modulated Communications Signals
Note 29	Modified Periodogram
Note 30	Bartlett's Periodogram
Note 31	Welch's Periodogram

PART VI FIR Filter Design

Note 32	Designing FIR Filters: Background and Options	1
Note 33	Linear-Phase FIR Filters	1
Note 34	Periodicities in Linear-Phase FIR Responses	1
Note 35	Designing FIR Filters: Basic Window Method	1
Note 36	Designing FIR Filters: Kaiser Window Method	1
Note 37	Designing FIR Filters: Parks-McClellan Algorithm	1

PART V Analog Prototype Filters

Note 38	Laplace Transform	-1
Note 39	Characterizing Analog Filters	-1
Note 40	Butterworth Filters	-1
Note 41	Chebyshev Filters	-1
Note 42	Elliptic Filters	-1
Note 43	Bessel Filters	-1

PART VI *z*-Transform Analysis

Note 44	The <i>z</i> Transform
Note 45	Computing the Inverse z Transform Using the Partial Fraction Expansion
Note 46	Inverse <i>z</i> Transform via Partial Fraction Expansion Case 1: All Poles Distinct with <i>M</i> < <i>N</i> in System Function
Note 47	Inverse <i>z</i> Transform via Partial Fraction Expansion Case 2: All Poles Distinct with $M \ge N$ in System Function (Explicit Approach)
Note 48	Inverse <i>z</i> Transform via Partial Fraction Expansion Case 3: All Poles Distinct with $M \ge N$ in System Function (Implicit Approach)

PART VII IIR Filter Design

Note 49	Designing IIR Filters: Background and Options	9-1
Note 50	Designing IIR Filters: Impulse Invariance Method	0-1
Note 51	Designing IIR Filters: Bilinear Transformation	1-1

PART VIII Multirate Signal Processing

Note 52	Decimation: The Fundamentals
Note 53	Multistage Decimators
Note 54	Polyphase Decimators
Note 55	Interpolation Fundamentals
Note 56	Multistage Interpolation
Note 57	Polyphase Interpolators

PART IX Bandpass and Quadrature Techniques

Note 58	Sampling Bandpass Signals
Note 59	Bandpass Sampling: Wedge Diagrams
Note 60	Complex and Analytic Signals
Note 61	Generating Analytic Signals with FIR Hilbert Transformers
Note 62	Generating Analytic Signals with Frequency-Shifted FIR Lowpass Filters
Note 63	IIR Phase-Splitting Networks for Generating Analytic Signals
Note 64	Generating Analytic Signals with Complex Equiripple FIR Filters
Note 65	Generating I and Q Channels Digitally: Rader's Approach
Note 66	Generating I and Q Channels Digitally: Generalization of Rader's Approach

PART X Statistical Signal Processing

Note 67	Parametric Modeling of Discrete-Time Signals	-1
Note 68	Autoregressive Signal Models	-1
Note 69	Fitting AR Models to Stochastic Signals: Yule-Walker Method	-1
Note 70	Fitting All-Pole Models to Deterministic Signals: Autocorrelation Method	-1
Note 71	Fitting All-Pole Models to Deterministic Signals: Covariance Method	-1
Note 72	Autoregressive Processes and Linear Prediction Analysis	-1
Note 73	Estimating Coefficients for Autoregressive Models: Burg Algorithm	-1
Index		-1

Preface

S tandard advice for writing a preface tells the author to begin by answering the question, "Why did you write this book?" The published answers almost always include an explanation of how something is still missing in the already vast body of existing literature, and how the book in question represents a valiant attempt to fill the void at least partially.

This book is no exception. There still is a dearth of *good* collections of step-bystep procedures, or *recipes*, for design and implementation of anything beyond just the most elementary DSP procedures. This book *is* an attempt to fill this void—at least partially. However, the tagline for this book is most definitely not meant to be, "Get a result without really gaining much understanding along the way." Here, the focus is clearly on the recipes, but supporting explanations and mathematical material are also provided. This supporting material is set off in such a way so that it is easily bypassed if the reader so desires.

This book provides an opportunity to delve deeper into the nuances of certain interesting topics within DSP. A good alternative title might be *Exploring the Nooks and Crannies of Digital Signal Processing*. As with all books, every reader will not resonate with every topic, but I'm confident that each reader will share an interest in a large subset of the topics presented.

Note 1, Navigating the DSP Landscape, provides diagrams that map the relationships among all the book's various topics. One diagram is dedicated to processing techniques that operate on real-valued digital signals to modify in some way the properties of those signals while leaving their fundamental real-valued and digital natures intact. A second diagram is dedicated to processing techniques that are concerned primarily with conversion between real-valued digital signals and other entities such as analog signals, complex-valued signals, and estimated spectra.

Many of the Notes include examples that demonstrate an actual application of the technique being presented. Most sections use MATLAB tools for routine tasks such as designing the digital filters that are used in the reference designs. When appropriate, the

use of these tools is discussed in the text. The results provided in Note 66, Generating I and Q Channels Digitally: Generalization of Rader's Approach, were generated by a modified version of the PracSim simulation package that is described in *Simulating Wireless Communication Systems* (Prentice Hall, 2004). However, the filter coefficients used in the simulation were generated using MATLAB. The examples make heavy use of MATLAB as a convenience. However, it is not my intent to make this a MATLAB "workbook" with projects and exercises that *require* the reader to use MATLAB, because I want the book to remain useful and attractive to readers who do not have access to MATLAB. The m-files for the MATLAB programs discussed in the book, as well as for programs used to generate some of the illustrations, can be found at the website www.informit. com/ph.

This book is not the best choice for a first book from which to learn DSP if you're starting from scratch. For this task, I recommend *Understanding Digital Signal Processing* by Richard Lyons (Prentice Hall, 2004). This book is, however, a good *N*+1st book for anyone—from novice to expert—with an interest in DSP. Its contents comprise an assortment of interesting tidbits, unique insights, alternative viewpoints, and rarely published techniques. The following are some examples.

- The set of five techniques for generating analytic signals presented in Notes 60 through 64 do not appear together in any single text.
- The visualization techniques used in Note 22 probably are not discussed anywhere else, because I came up with them while writing this book. These techniques follow directly from first principles, but I've never seen them explicitly presented elsewhere.
- Natural sampling, as discussed in Note 6, usually can be found only in older texts that cover traditional (that is, analog) communication theory.

My overarching goal was to write an easy-to-read book loaded with easy-to-access information and easy-to-use recipes. I hope I have succeeded. This page intentionally left blank

Navigating the DSP Landscape

D igital signal processing (DSP) is based on the notions that an analog signal can be digitized and that mathematical operations can effectively take the place of (or even surpass) electronic manipulations performed on the original analog signal. In the earliest days of DSP, its applications were limited to sonar and seismology because these fields utilized low-bandwidth signals that could be sampled at adequate rates using the available technology. As digital processing circuits and analog to-digital converters have become faster and faster, the number of applications for DSP has exploded.

Hundreds of techniques (and variations thereof) are used in DSP, and it can be difficult to see the big picture—how all these various techniques relate to each other and to a particular application at hand. Rather than a comprehensive treatment of all the possible topics within DSP, this book is an attempt to document in-depth explorations of some of the "nooks and crannies" in DSP that often are glossed over in traditional texts. Figures 1.1 and 1.2 show diagramatically the realtionships among the various processing techniques explored in this book. The topic areas are arbitrarily split into two groups



Figure 1.1 Processing techniques that modify the properties of real-valued digital signals. The numbers "Nnn" indicate the Notes in which each technique is discussed. Solid paths indicate "run-time" data connections. Dashed paths indicate "design-time" connections.

for organizational purposes. Figure 1.1 shows those techniques that are concerned primarily with operating on real-valued digital signals to modify in some way the properties of those signals while leaving their fundamental real-valued and digital natures intact. Given that complex-valued signals are really just quadrature pairs of real-valued signals, most of these techniques are easily extended to corresponding complex cases. Figure 1.2 shows those techniques that are concerned primarily with conversion between real-valued digital signals and other entities such as analog signals, complexvalued signals, and spectrum estimates.



Figure 1.2 Processing techniques that convert real-valued digital signals to or from other things such as analog signals, complex-valued digital signals, or spectral estimates. The numbers "Nnn" indicate the Notes in which each technique is discussed. Solid paths indicate "run-time" data connections. Dashed paths indicate "design-time" connections.

Overview of Sampling Techniques

This note discusses the difference between *implicit* and *explicit* sampling. It introduces three different mathematical models of explicit sampling techniques—*ideal* sampling, *natural* sampling, and *instantaneous* sampling.

Digitization of an analog signal is the one process above all others that makes DSP such a useful technology. If it were limited to working with only those signals that orginate in digital form, DSP would be just an academic curiosity. Digitization actually comprises two distinct operations: sampling and quantization, which are usually analyzed separately.

2.1 Implicit Sampling Techniques

In *implicit sampling*, a sample measurement is triggered by the signal attaining some specified value or crossing some specified threshold. Recording the times at which zero-crossings occur in a bipolar signal is an example of implict sampling.

2.2 Explicit Sampling Techniques

Unlike implict sampling, in which samples are triggered by some aspect of signal behavior, in *explicit sampling*, signal values are measured at specified times without regard to the signal's behavior. Consider the continuous-time sinusoidal signal and its two-sided magnitude spectrum depicted in Figure 2.1. There are three explicit sampling techniques *natural sampling, instantaneous sampling*, and *ideal sampling*—that can be used to sample such a signal. The results produced by these techniques, and the corresponding impacts on the signal's spectrum are compared in Key Concept 2.1.

Ideal Sampling

As depicted in Key Concept 2.1, zero-width samples take on instantaneous values of the analog signal. Neglecting quantization and timing errors, the sequence of values produced by an analog-todigital converter can be modeled as the output of



Figure 2.1 Continuous-time sinusoid and its two-sided magnitude spectrum

an ideal sampling process. Ideal sampling is discussed further in Note 3.

Natural Sampling

Nonzero-width samples have time-varying amplitudes that follow the contours of the analog signal, as shown in Key Concept 2.1. Commutator systems for timedivision multiplexing of telegraph signals, first proposed in 1848, used an approximation to natural sampling. The sample pulses were created by gating a signal with rotating mechanical contacts. This multiplexing technique was subsequently applied to telephone signals in 1891. It was the application of natural sampling to telephony that first led to consideration of just how rapidly a continuous-time signal needed to be sampled in order to preserve fidelity and ensure the ability to reconstruct exactly the original, unsampled signal. Natural sampling is explored further in Note 6.

Instantaneous Sampling

In instantaneous sampling, nonzero-width samples each have a constant amplitude that corresponds to the instantaneous value of the analog signal at the beginning of the sample. The sample values are held constant long enough to create flat-topped sample pulses. The output of a digital-toanalog converter (DAC) can be modeled as the output of an instantaneous sampling process, often as the limiting case in which the sample width equals the sampling interval. As discussed in Note 7, the results of the instantaneous sampling model play a key role in the specification of the analog filter used to smooth the DAC output.



Reference

1. H. D. Lüke, "The Origins of the Sampling Theorem," *IEEE Communications*, April 1999, pp. 106–108. This page intentionally left blank

Index

3-point DFT algorithm, 19-16-dB bandwidth of bin response in periodograms, 26-27-point DFT algorithm, 19-3

A

ACS (autocorrelation sequence), 68-1 ADC (analog-to-digital) converters, 4-1 Additive systems, 39-2 Additive white Gaussian noise (AWGN), 27-1 to 27-5 Additivity property Fourier series, 10-2 Fourier transform, 11-3 Laplace transform, 38-2 z transform, 44-4 Aliased sinc function, 14-3 Aliasing ideal sampling, 3-2 to 3-3, 4-1 to 4-2 impulse invariance method, 50-1 to 50-2 All-pole (AP) models deterministic signals, 70-1 to 70-2, 71-1 to 71-3 discrete-time signals, 67-1 to 67-2, 67-5 All-zero (AZ) models, 67-3, 67-5 All-zero filters, 32-1 Almost-all-pass ASG filters, 61-1 to 61-2 Almost-all-positive pass ASG filters, 62-1 to 62-2 Amplitude-phase form in linear-phase FIR filters, 34-1 to 34-4 Analog filters bandpass transformations, 39-6 lowpass response, 39-4 to 39-5 magnitude, phase, and delay responses, 39-3 to 39-4 overview, 39-1 to 39-2 passband transformations, 39-5 to 39-7 transfer functions, 39-2 to 39-3 Analog-to-digital (ADC) converters, 4-1 Analysis DFT, 13-2 Analytic associates, 60-1 Analytic-like signals, 60-2 Analytic signal generation (ASG) filters bandpass, 21-1 designing, 62-1 to 62-3 Hilbert transformers, 61-1 ideal response characteristics, 64-1 Analytic signals, 60-1 to 60-2 complex equiripple FIR filters for, 64-1 to 64-3 discrete-time, 60-2 to 60-3

frequency-shifted FIR lowpass filters for, 62-1 to 62-3 generating, 60-3 to 60-6 Hilbert transformers for, 61-1 to 61-3 IIR phase-splitting networks, 63-1 to 63-7 via spectrum tailoring, 60-4 Anti-aliasing filters decimators, 52-1 to 52-3 ideal sampling, 4-1 to 4-2 Anti-imaging filters, 55-1, 55-4, 56-1 to 56-4 Antoniou, A., 25-1 AP (all-pole) models deterministic signals, 70-1 to 70-2, 71-1 to 71-3 discrete-time signals, 67-1 to 67-2, 67-5 AR (autoregressive) models. See Autoregressive (AR) modeling. ARMA (autoregressive-moving-average) models, 67-1 to 67-5 ASG (analytic signal generators) bandpass, 21-1 designing, 62-1 to 62-3 Hilbert transformers, 61-1 ideal response characteristics, 64-1 Attenuation of side lobes, 23-2 Audio CD player sample rates, 8-3 to 8-4 Autocorrelation method all-poles models, 70-1 to 70-2 Yule-Walker method, 69-1 Autocorrelation sequence (ACS), 68-1 Autoregressive (AR) modeling coefficient estimates, 73-1 to 73-3 and linear prediction analysis, 72-1 to 72-5 overview, 68-1 to 68-2 parametric modeling, 67-1 to 67-3, 67-5 stochastic signals, 69-1 to 69-2 Autoregressive-moving-average (ARMA) models, 67-1 to 67-5 AWGN (additive white Gaussian noise), 27-1 to 27-5 AZ (all-zero) models, 67-3, 67-5

B

Backward linear prediction, 72-2 to 72-3 Bandpass filters equiripple, 64-2 FIR filter approximation, 35-2 to 35-3 Hilbert transformers for, 61-3 transformations, 39-6 Bandpass sampling, use of wedge diagrams, 59-1 to 59-4 Bandpass signals, sampling of, 58-1 to 58-3 Bandstop filters FIR filter approximation, 35-2 to 35-3 transformations, 39-6 to 39-7 bartlett function, 21-2 Bartlett windows, 21-1 to 21-2 Bartlett's periodogram, 30-1 to 30-3 Basic window method for FIR filters, 35-1 Bennett, W. R., 5-4 Bessel filters, 43-1 to 43-2 Bias in periodograms, 26-2 **Biased** estimates autocorrelation method, 70-2 Yule-Walker method, 69-2 Bilateral z-transform pairs, 44-2 bilinear function, 51-5 Bilinear transformations, 51-1 Butterworth filters, 40-2 MATLAB for, 51-5 prewarping, 51-1 to 51-3 Bin-centric approach for window analysis, 22-1 to 22-2 Bin numbers in DFT, 15-3 Bin response for periodograms, 26-2 Bit-reversed order, 17-3 Blackman windows, 21-1, 25-2 to 25-4 Boxcar FIR averaging filters, 34-1, 34-3 Burg algorithm, 73-1 to 73-3 butter function, 51-5 Butterflies, 17-3 to 17-5 Butterworth filters, 39-1, 40-1 to 40-2 frequency response, 40-2 to 40-3 lowpass response, 39-5 prototypes, 40-2, 40-4 to 40-5

С

C/D (continuous-to-discrete) converters, 3-1, 5-3 Carrier delay in analog filters, 39-4 Cascade structure for even-length FIR filters, 32-3 to 32-4 Cauchy's residue theorem, 45-3 Causal systems, 39-2 CDs (compact discs) DAT conversions, 56-3 sample rates, 8-3 to 8-4 Ceiling function, 35-1 cfirpm function, 64-1 to 64-2 CGD (constant group delay) filters, 33-1 to 33-4 cheblord function, 41-3 Chebyshev filters, 39-1

lowpass response, 39-5 overview, 41-1 to 41-3 prototypes, 41-4 renormalizing, 41-2 Chebyshev polynomials, 41-1 Cholesky decomposition, 71-2 to 71-3 Classical form of Fourier series, 10-1 Coefficient estimates in autoregressive models, 73-1 to 73-3 Coefficient matching approach in inverse z transform, 46-1 to 46-3 Coherent gain, 23-3 Comb function, 5-2 Commutator systems, 2-2 Compact discs (CDs) DAT conversions, 56-3 sample rates, 8-3 to 8-4 Complete elliptic integrals, 63-1, 63-3 Complex conjugates of analytic signals, 60-1 Complex equiripple FIR filters, 64-1 to 64-3 Complex filter approach for generating analytic signals, 60-6 Computational burden of multistage interpolation, 56-3 Conjugate-analytic signals, 60-1 Conjugation property Fourier transform, 11-3 z transform, 44-4 Constant group delay (CGD) filters, 33-1 to 33-4 Continuous-phase frequency shift keyed (CPFSK) signals periodogram performance, 28-1 to 28-2, 29-2, 30-3, 31-3 power spectral density for, 28-1 Continuous-time Fourier transform (CTFT), 5-1, 11-1 Continuous-time Kaiser windows, 25-2 Continuous-to-discrete (C/D) converters, 3-1, 5-3 Convolution, fast, 20-1 to 20-3 Convolution property discrete-time Fourier transform, 12-2 Fourier series, 10-2 Fourier transform, 11-3 Laplace transform, 38-2 z transform, 44-4 Cosine modulation property, 11-3 Covariance method, 71-1 to 71-3 CPFSK. See continuous-phase frequency shift keyed signals. cremez function, 64-1 Critically sampled signals, 8-3 CTFT (continuous-time Fourier transform), 5-1, 11-1

D

DACs (digital-to-analog converters) modeled as instantaneous sampling, 2-2 signal reconstruction, 8-1 to 8-4 DAT (digital audio tape) signals, 56-3 Data windows Hann, 24-5 Kaiser, 25-3 Welch's periodograms, 31-1 Decimation and decimators efficient FIR decimators, 52-3 to 52-4 multistage, 53-1 to 53-3 overview, 52-1 to 52-3 polyphase, 54-1 to 54-2 Decimation-in-frequency algorithms, 18-1 to 18-2 Decimation-in-time, natural input-permuted output (DIT-NIPO) FFT, 17-3 Decimation-in-time, permuted input-natural output (DIT-PINO) FFT, 17-3 Decimation-in-time algorithms, 17-1 to 17-5 Delav analog filters, 39-3 to 39-4 Bessel filters, 43-1 linear-phase filters, 33-1 Delta functions comb. 5-2 Dirac, 5-1 to 5-2 overview, 5-1 sampling model, 5-3 Deterministic signals, fitting all-pole models to autocorrelation method, 70-1 to 70-2 covariance method, 71-1 to 71-3 DFT. See Discrete Fourier transform (DFT). Difference equation autoregressive signal models, 68-2 IIR filters, 49-2 to 49-3 Differentiation property discrete-time Fourier transform, 12-2 Fourier transform, 11-3 z transform, 44-4 Digital audio tape (DAT) signals, 56-3 Digital signal processing (DSP) overview, 1-1 to 1-2 Digital-to-analog converters (DACs) instantaneous sampling, 2-2 signal reconstruction, 8-1 to 8-4 Dirac, Paul, 5-1 Dirac combs, 5-2 Dirac delta function overview, 5-1 to 5-2 sampling model, 5-3

Direct form decimators, 52-3 to 52-4 FIR structure, 32-2, 32-4 to 32-5 IIR filters, 49-1, 49-3 to 49-4 interpolators, 55-4 to 55-5 diric function, 14-1 to 14-2 Dirichlet, Peter Gustav Lejeune, 10-2 Dirichlet conditions in Fourier series, 10-2 Dirichlet kernel, 14-2 to 14-3, 27-3 to 27-4 Discrete Fourier transform (DFT), 9-2 deriving from DTFT, 13-3 even-length windows for, 21-3 to 21-4 leakage, 13-4, 15-1 to 15-3 lengthening, 16-4 overview, 13-1 to 13-2 periodicity in frequency domain, 13-2 to 13-3 periodicity in time domain, 13-4 plotting with DTFT on same graph, 15-3 prime factor algorithms, 19-1 to 19-3 resolution, 16-1 to 16-4 sampling theorem, 5-1 synthesis, 13-2 truncation, 14-2 Discrete-time Fourier transform (DTFT), 11-1 deriving, 12-1 description, 9-2 DFT derived from, 13-3 pairs, 12-2 plotting with DFT on same graph, 15-3 properties, 12-2 rectangular windows, 14-2 to 14-3 sampling theorem, 5-1 sinusoidal pulses, 15-2 to 15-3 truncation, 14-2 Discrete-time Kaiser windows, 25-3 Discrete-time signals analytic, 60-2 to 60-3 parametric modeling, 67-1 to 67-5 properties, 60-4 DIT-NIPO (decimation-in-time, natural inputpermuted output) FFT, 17-3 DIT-PINO (decimation-in-time, permuted inputnatural output) FFT, 17-3 Dolph-Chebyshev windows, 24-3 to 24-4 Downsamplers efficient FIT decimators, 52-3 to 52-4 IIR phase-splitting networks, 63-1 multistage decimators, 53-1 polyphase decimators, 54-1 to 54-2 DTFT. See Discrete-time Fourier transform (DTFT). Duality property in Fourier transform, 11-3

E

Elementary windows, 21-1 to 21-3 ellipj function, 63-3, 63-6 ellipke function, 63-3 to 63-4, 63-6 Elliptic filters, 39-1, 42-1 to 42-2 lowpass response, 39-5 minimum required order, 42-2 prototypes, 42-3 Elliptic phase-splitting networks, 63-3 Envelope delay in analog filters, 39-4 Equiripple filters analytic signals, 64-1 to 64-3 Parks-McClellan algorithm, 37-1 Equivalent noise bandwidth, 23-1 to 23-3 Error filters in linear prediction, 72-2 to 72-3 Even-length windows for DFT applications, 21-3 to 21-4 Explicit IIR filters, 49-2 Explicit sampling techniques, 2-1 ideal sampling, 2-1 to 2-2 instantaneous sampling, 2-2 natural sampling, 2-2

F

Fast convolution, 20-1 to 20-3 Fast Fourier transform (FFT), 9-2, 13-1 to 13-2 decimation-in-frequency algorithms, 18-1 to 18-2 decimation-in-time algorithms, 17-1 to 17-5 fast convolution using, 20-1 to 20-3 prime factor algorithm, 19-1 to 19-3 Feature mapping in impulse invariance method, 50-2 to 50-4 Fejer kernel, 27-3 FFT. See Fast Fourier transform (FFT). Filters analog. See Analog filters. anti-aliasing, 4-1 to 4-2, 52-1 to 52-3 anti-imaging, 55-1, 55-4, 56-1 to 56-4 bandpass. See Bandpass filters. bandstop, 35-2 to 35-3, 39-6 to 39-7 Bessel, 43-1 to 43-2 Butterworth. See Butterworth filters. Chebyshev. See Chebyshev filters. elliptic, 42-1 to 42-3 equiripple filters, 37-1, 64-1 to 64-3 FIR. See Finite-impulse-response (FIR) filters. IIR. See Infinite impulse response (IIR) filters. Finite-impulse-response (FIR) filters background and options, 32-1 basic window method, 35-1

decimation, 52-3 to 52-4, 53-1 Fourier series for, 10-1 implementation structures, 32-2 to 32-5 Kaiser window method, 36-1 to 36-3 linear-phase, 32-1, 32-3, 33-1 to 33-4, 34-1 to 34-4 Parks-McClellan algorithm, 37-1 to 37-2 prediction error filters, 72-1 FIR Hilbert transformers, 61-1 to 61-3 firpmord function, 53-3, 56-3 First derivative property for Laplace transform, 38-2 First negative Nyquist zones, 60-2 First positive Nyquist zones, 60-2 5-point DFT algorithm, 19-2 Fixed systems, 39-2 Flat-topped sampling, 7-1 Folding frequency decimators, 53-1 ideal sampling, 3-3 Forward linear prediction, 72-1 Fourier analysis overview, 9-1 categories, 9-1 DFT, 9-2 DTFT, 9-2 FFT. 9-2 Fourier series, 9-1 Fourier transform, 9-2 Fourier series (FS), 9-1, 10-1 classical form, 10-1 Dirichlet conditions, 10-2 FIR filters, 35-1 modern form, 10-1 to 10-2 Fourier transform (FT), 9-2, 11-1 pairs, 11-2 properties, 11-3 Frequency mapping for bilinear transformation, 51-1 to 51-2 Frequency response analog filters, 39-3 bandpass filters, 64-3 Bessel filters, 43-1 to 43-2 bilinear transformation, 51-3 to 51-4 Butterworth filters, 40-2 to 40-3 Chebyshev filters, 41-3 elementary windows, 21-1 Hilbert transformers, 61-2 to 61-3 ideal digital filters, 35-1, 35-3 IIR filters, 49-2 to 49-3, 50-1, 51-4 Kaiser windows, 25-1 linear-phase FIR filters, 33-2, 34-1 lobe structure in, 21-4 lowpass filters, 62-1 to 62-3

Frequency shift property DTFT, 12-2 Fourier series, 10-2 Fourier transform, 11-3 Laplace transform, 38-2 z transform, 44-4 freqz function, 51-3, 63-4, 63-6

G

Gain, processing, 23-3 to 23-4 Gaussian noise, 27-1 to 27-5 Gibb's phenomenon, 21-1 Gold, B., 41-2 to 41-3, 63-1 Group delay analog filters, 39-3 Bessel filters, 43-1 linear-phase FIR filters, 33-1 Guard bands in wedge diagrams, 59-4

Η

Hamming windows, 24-3, 24-5 Hann windows, 21-1, 24-3, 24-5 Hat notation, 72-1 Hermitian matrix, 71-2 to 71-3 Highpass filters approximation, 35-2 to 35-3 hilbert function, 60-5 Hilbert transformers, 60-2 analytic signals, 60-5, 61-1 to 61-3 linear-phase FIR filters, 33-1 Homogeneity property Fourier series, 10-2 Fourier transform, 11-3 Laplace transform, 38-2 z transform, 44-4 Homogeneous systems, 39-2

I

I (inphase) channel digital generation Rader approach generalization, 66-1 to 66-5 Rader approach overview, 65-1 to 65-4 Ideal samplers, 3-1 Ideal sampling aliasing, 3-2 to 3-3 description, 2-1 to 2-2 overview, 3-1 to 3-2 practical application, 4-1 to 4-2 IF (intermediate-frequency) signals, 60-1 IIR. *See* Infinite impulse response (IIR) filters. IIR phase-splitting networks, 63-1 to 63-7 IIR sample response in inverse z transform, 45-1 Images in ideal sampling, 3-2 Imaginary part property for Fourier transform, 11-3 Implicit sampling techniques, 2-1 Impulse invariance method, 50-1 aliasing, 50-1 to 50-2 direct, 50-2 feature mapping, 50-2 to 50-4 Impulse response analog filters, 39-1 to 39-2 linear-phase FIR filters, 33-1 to 33-4 Infinite impulse response (IIR) filters background and options, 49-1 to 49-4 bilinear transformation, 51-1 to 51-5 implementation structures, 49-3 to 49-4 impulse invariance method, 50-1 to 50-4 inverse z transform, 45-2 Inphase (I) channel digital generation Rader approach generalization, 66-1 to 66-5 Rader approach overview, 65-1 to 65-4 Instantaneous sampling, 2-2, 7-1 to 7-3 Integration property Fourier transform, 11-3 Laplace transform, 38-2 Interpolation, 55-1 anti-imaging filters, 55-4 efficient structures, 55-4 to 55-5 multistage, 56-1 to 56-4 polyphase, 57-1 to 57-2 upsampling, 55-1 to 55-3 Inverse DFT, 13-2 Inverse Laplace transform, 38-1 Inverse Levinson recursion, 72-3 Inverse transform, 11-1 Inverse z transform using partial fraction expansion, 45-1 to 45-3 all poles distinct with M < N in system function, 46-1 to 46-3 all poles distinct with $M \ge N$ in system function, 47-1 to 47-2, 48-1 to 48-3

J-K

Jacobian elliptic function, 63-1, 63-3 to 63-7 Kaiser windows characteristics, 24-3 to 24-5 FIR filters, 36-1 to 36-3 working with, 25-1 to 25-4 Kaiser-Bessel windows, 25-1 to 25-2 Kay, M., 44-1 Kotelnikov, A., 5-4 *k*th derivative property for Laplace transform, 38-2

L

Lag windows description, 21-1 Hann, 24-5 Laplace transform, 38-1 impulse invariance method, 50-2 pairs and properties, 38-2 transfer functions, 39-2 Lattice implementation in linear prediction, 72-2 to 72-4 Leakage DFT, 13-4, 15-1 to 15-3 even-length windows for, 21-3 to 21-4 from truncation, 14-1 Lengthening DFTs, 16-4 Levinson recursion ARMA model, 67-4 autocorrelation method, 70-1 to 70-2 coefficient estimates, 73-1 Yule-Walker method, 69-1 to 69-2 Levinson-Durbin recursion autocorrelation method, 70-1 covariance method, 71-2 to 71-3 Line splitting, 73-3 Linear-phase FIR filters, 32-1, 32-3, 33-1 to 33-2 impulse response, 33-4 periodicities in, 34-1 to 34-4 properties, 33-3 Linear prediction analysis, 72-1 to 72-5 Linear property for DTFT, 12-2 Linear systems in analog filters, 39-2 Linearity property Fourier series, 10-2 Fourier transform, 11-3 Laplace transform, 38-2 z transform, 44-4 Lobes attenuation, 23-2 in frequency response, 21-4 Kaiser windows, 25-2 modified periodograms, 29-1 to 29-2 Welch's periodograms, 31-1 width, 23-1 Loss, scalloping, 23-4 Lowpass filters Bessel, 43-1 to 43-2 Butterworth, 40-1 to 40-5 Chebyshev, 41-1 to 41-3 FIR filters approximation, 35-1, 35-3 FIR filters using Kaiser window, 36-1 Lowpass response of analog filters, 39-4 to 39-5 Lüke, H. D., 5-4

Μ

MA (moving-average) models, 67-1, 67-5 Magnitude-phase form for linear-phase FIR filters, 34-1 Magnitude response analog filters, 39-3 to 39-6 Bessel filters, 43-2 bilinear transformation, 51-3 to 51-4 Butterworth filters, 40-1, 40-3 Chebyshev filters, 41-1 to 41-3 Dolph-Chebyshev windows, 24-4 elliptic filters, 140 FIR filters, 37-2 rectangular windows, 23-1, 24-2 triangular windows, 24-3 Magnitude spectrum instantaneous sampling, 7-2 to 7-3 natural sampling, 6-2 to 6-3 Main lobe width, 23-1 Marple, S. L., 60-2, 71-3 Mathematical convention for IIR filters, 49-2 Maximum entropy method (MEM), 73-1 Maximum entropy spectrum analysis (MESA), 73-1 McClellan, John H., 37-1, 71-3 MEM (maximum entropy method), 73-1 MESA (maximum entropy spectrum analysis), 73-1 Minimum-phase FIR design, 33-1 Modified periodograms, 27-5, 29-1 to 29-3 Morf, M., 71-3 Moving-average filters, 32-1 Moving-average (MA) models, 67-1, 67-5 Moving-average process, 67-5 Multiplexing, 2-2, 5-4 Multiplication property Fourier series, 10-2 Fourier transform, 11-3 Multistage decimators, 53-1 to 53-3 Multistage interpolation, 56-1 to 56-4

Ν

Natural sampling, 2-2, 6-1 to 6-3 Negative-like discrete-time analytic signals, 60-2 Neper frequency, 38-1 Noise AWGN, 27-1 to 27-5 Bartlett's periodogram, 30-1 equivalent noise bandwidth, 23-1 to 23-3 ideal sampling, 4-2 Nonuniform sampling, 3-1 Normalized frequency in linear-phase FIR filters, 33-2 Nyquist, H., 5-4 Nyquist bandwidth in signal reconstruction, 8-3 to 8-4 Nyquist zones in discrete-time analytic signals, 60-2 to 60-3

0

Observability, zero-padding for, 16-2 One-sided *z* transform, 44-1 Oppenheim, A. V., 3-1 Overlap-and-save fast convolution, 21-1 to 21-2 Oversampling CD players, 8-4

P

Paired-filter approach for analytic signals, 60-5 to 60-6 Parametric modeling, 67-1 to 67-5 Parks, Thomas, 37-1 Parks-McClellan (PM) algorithm ASG filters, 62-1 FIR filters, 37-1 to 37-2 Hilbert transformers, 61-2 Parseval's theorem, 23-3, 26-2 Partial fraction expansion (PFE) for inverse z transform, 45-1 to 45-3 all poles distinct with M < N in system function, 46-1 to 46-3 all poles distinct with $M \ge N$ in system function, 47-1 to 47-2, 48-1 to 48-3 Passband ASG filters, 64-1 lowpass analog filters, 39-4 to 39-5 with phase-splitters, 63-4, 63-6 Passband transformations, 39-5 to 39-7 Periodicities in frequency domain, 13-2 to 13-3 linear-phase FIR filters, 34-1 to 34-4 in time domain, 13-4 Periodograms Bartlett's, 30-1 to 30-3 modified, 27-5, 29-1 to 29-3 modulated communications signals, 28-1 to 28-2 sinusoids in AWGN, 27-1 to 27-5 unmodified, 26-1 to 26-2 Welch's, 31-1 to 31-4 PFA (prime factor algorithm), 19-1 to 19-3 Phase delay in analog filters, 39-4 Phase response analog filters, 39-3 to 39-4 Bessel filters, 43-2 bilinear transformation, 51-3 to 51-4 Butterworth filters, 40-3 Chebyshev filters, 41-3 FIR filters, 32-1

linear-phase FIR filters, 33-1 to 33-3, 34-1 to 34-4 paired-filter approach, 60-5 to 60-6 Rader's approach, 66-4 Phase-splitting networks, 63-1 to 63-7 Physical signal reconstruction, 8-1 to 8-4 Picket fence effect, 16-1 Plotting DFT and DTFT on same graph, 15-3 PM (Parks-McClellan) algorithm ASG filters, 62-1 FIR filters, 37-1 to 37-2 Hilbert transformers, 61-2 Pole-zero (PZ) models, 67-1, 67-3 to 67-5 Poles Butterworth filters, 40-1 to 40-2 elliptic filters, 42-2 IIR filters, 49-3 to 49-4 inverse z transform, 45-3 transfer functions, 39-3 z transform, 44-1 Pollak, H., 25-1 poly function, 51-5 Polynomial division for inverse z transform, 47-2 Polynomials, Chebyshev, 41-1 Polyphase decimators, 54-1 to 54-2 Polyphase interpolators, 57-1 to 57-2 Positive-like discrete-time analytic signals, 60-2 PracSim simulation, 28-1, 66-1 Prediction analysis, 72-1 to 72-5 Prediction error filters, 72-1 Prewarped frequencies bilinear transformation, 51-1 to 51-3 Butterworth filters, 40-2, 40-4 elliptic filters, 42-2 Prime factor algorithm (PFA), 19-1 to 19-3 Processing gain of windows, 23-3 to 23-4 Prolate spheroidal wave functions, 25-1 Prony method, 71-1 Proper rational function in inverse z transform, 46-1 Prototypes Butterworth filters, 40-2, 40-4 to 40-5 Chebyshev filters, 41-4 elliptic filters, 42-3 PSD (power spectral density) autoregressive signal models, 68-2 CPFSK, 28-1 I and Q channels digital generation, 66-3 PZ (pole-zero) models, 67-1, 67-3 to 67-5

Q

Q (quadrature) channels digital generation Rader approach generalization, 66-1 to 66-5 Q (quadrature) channels digital generation, (*continued*) Rader approach overview, 65-1 to 65-4 Quotients in inverse *z* transform, 47-2

R

Raabe, H., 5-4 Rabiner, L. R., 41-2 to 41-3 Rader, C. M. I and Q channels digital generation generalization, 66-1 to 66-5 Rader, C. M., (continued) I and Q channels digital generation overview, 65-1 to 65-4 phase-splitting networks, 63-1 to 63-2, 63-4 Radian frequency in Laplace transforms, 38-1 Real part property for Fourier transform, 11-3 Realizable systems in analog filters, 39-3 Reconstruction filters, 8-1 to 8-4 Rectangular windows characteristics, 24-1 to 24-3 description, 21-1 DTFT, 14-2 to 14-3 Region of convergence (ROC) in z transform, 44-1 to 44-2 Reilly, A., 60-5, 62-1 Relaxed systems, 39-2 Remez algorithm, 37-1 Remez exchange, 37-1 Renormalizing Bessel filters, 43-1 to 43-2 Chebyshev filter transfer functions, 41-2 Residues in inverse z transform, 45-3 Resolution DFT, 16-1 to 16-4 lengthening for, 16-4 sinusoids in AWGN, 27-4 zero-padding for, 16-3 ROC (region of convergence) in z transform, 44-1 to 44-2

S

Sample spectrum, 26-2 Sampling, 2-1 bandpass signals, 58-1 to 58-3 decimators, 52-1, 53-2 delta functions, 5-1 to 5-3 explicit, 2-1 to 2-2 ideal, 3-1 to 3-3, 4-1 to 4-2 implicit, 2-1 instantaneous, 7-1 to 7-3 interpolation, 55-1 natural, 6-1 to 6-3

wedge diagrams, 59-1 to 59-4 Sampling jitter, 3-1 Sampling theorem, 5-4 Scaling, frequency, 16-3 Scalloping loss, 23-4 Schafer, R. W., 3-1 Schwartz, Laurent, 5-2 Scott, N. L., 59-1 Second derivative property for Laplace transform, 38-2 Second Nyquist zones, 60-2 Second-order sections in FIR filters, 32-3 Selectivity factor in elliptic filters, 42-2 SFGs (signal flow graphs), 17-1 to 17-2 Shah function, 5-2 Shannon's sampling theorem, 5-4 Side lobes attenuation, 23-2 Kaiser windows, 25-2 modified periodograms, 29-1 to 29-2 Welch's periodograms, 31-1 Sifting property for Dirac delta function, 5-2 Signal-centric approach for window analysis, 22-1 to 22-4 Signal flow graphs (SFGs), 17-1 to 17-2 Signal-to-noise ratio (SNR) Bartlett's periodogram, 30-1 in processing gain, 23-3 sinusoids in AWGN, 27-1 to 27-2 Sinc envelopes in DAC converters, 8-1 Sine modulation property for Fourier transform, 11-3 Sinusoidal analysis techniques, 22-1 to 22-4 Sinusoidal pulses for DTFT, 15-2 to 15-3 Slepian, D., 25-1 Small-N transforms with prime factor algorithm, 19-1 SNR (signal-to-noise ratio) Bartlett's periodogram, 30-1 in processing gain, 23-3 sinusoids in AWGN, 27-1 to 27-2 SOI (signals of interest), 16-2 to 16-3 Spectral impacts in upsampling, 55-2 to 55-3 Spectral nulls in Welch's periodograms, 31-3 Spectrum tailoring approach for analytic signal generation, 60-3 to 60-4 Stability of analog filters, 39-3 Stationary systems, 39-2 Steady-state response in analog filters, 39-3 Step response in analog filters, 39-2 Stochastic signals, fitting AR models to, 69-1 to 69-2 Stopband ASG filters, 64-1 lowpass analog filters, 39-4 to 39-5 Stoppass frequency in elliptic filters, 42-1 to 42-2

Storer, J. E., 63-1 Synthesis DFT, 13-2 System functions autoregressive signal models, 68-2 IIR filters, 49-2 to 49-3 inverse *z* transform, 45-2 to 45-3 Systems convention for IIR filters, 49-2

Т

Telegraph signals, 2-2 Telephone signals, 2-2, 5-4 Time and frequency scaling property for Fourier transform, 11-3 Time-division multiplexing, 2-2 Time-invariant systems, 39-2 Time reversal property discrete-time Fourier transform, 12-2 z transform, 44-4 Time shift property discrete-time Fourier transform, 12-2 Fourier series, 10-2 Fourier transform, 11-3 Laplace transform, 38-2 z transform, 44-4 Timing errors in wedge diagrams, 59-3 to 59-4 Toeplitz matrix autocorrelation method, 70-1 covariance method, 71-3 Yule-Walker method, 69-1 Transfer functions analog filters, 39-2 to 39-3 Chebyshev filters, 41-1 to 41-2 lowpass Butterworth filters, 40-1 Transformations bandstop, 39-6 to 39-7 bilinear, 40-2, 51-1 to 51-5 Butterworth filters, 40-2 passband, 39-5 to 39-7 Transformers, Hilbert, 61-1 to 61-3 Transition bands ASG filters, 64-1 decimators, 53-1 to 53-2 lowpass analog filters, 39-4 to 39-5 Transposed direct form of FIR structure, 32-2 triang function, 21-2 Triangular windows bin-centric analysis approach, 22-2 characteristics, 24-2 to 24-3 description, 21-1 to 21-2 signal-centric analysis approach, 22-3 to 22-4 Truncation of signals, 14-1 to 14-3

Twiddle factors, 17-2 Two-sided *z* transform, 44-1 Two-stage decimators, 53-1 to 53-3 Type 1 linear-phase FIR filters, 33-1 to 33-3 Type 2 linear-phase FIR filters, 33-1 to 33-3 Type 3 linear-phase FIR filters, 33-2 to 33-3

U

Unbiased estimates autocorrelation method, 70-2 Yule-Walker method, 69-2 Uniform bandpass sampling, 58-1 Uniform sampling theorem, 5-4 Unilateral *z*-transform pairs, 44-2 Unit impulse in Dirac delta function, 5-2 Unit sample function, 49-2 to 49-3 Unmodified periodograms, 26-1 to 26-2 Upsampling interpolation, 55-1 to 55-3 spectral impacts, 55-2 to 55-3

V

Van Valkenberg, M. E., 41-3 Variance Bartlett's periodogram, 30-3 unmodified periodograms, 26-2 Vaughan, R. G, 59-1

W

Wedge diagrams, 59-1 to 59-2 interpreting, 59-2 to 59-3 timing errors, 59-3 to 59-4 Welch's periodograms, 31-1 to 31-4 White, D. L., 59-1 Windows Bartlett, 21-1 to 21-2 Blackman, 21-1, 25-2 to 25-4 DFT, 13-4 Dolph-Chebyshev, 24-4 DTFT, 14-2 to 14-3 elementary, 21-1 to 21-3 equivalent noise bandwidth, 23-2 to 23-3 even-length, 21-3 to 21-4 FIR filters, 35-1 to 35-3, 36-1 to 36-3 Hamming, 24-3, 24-5 Hann, 21-1, 24-3, 24-5 Kaiser. See Kaiser windows. lag, 21-1, 24-5 lobe structure in frequency response, 21-4 main lobe width, 23-1 processing gain, 23-3 to 23-4 rectangular. *See* Rectangular windows. scalloping loss, 23-4 side lobe attenuation, 23-2 sinusoidal analysis techniques for, 22-1 to 22-4 triangular. *See* Triangular windows. Worst case processing loss, 23-4

Y

Yule-Walker method and autocorrelation method, 70-1 autoregressive signal models, 68-1 stochastic signals, 69-1 to 69-2

Ζ

Z transform, 44-1 to 44-4 Zero-order-data-hold (ZODH) sampling, 7-1 Zero-padding, 16-1 to 16-3 Zeros elliptic filters, 42-2 IIR filters, 49-3 to 49-4 z transforms, 44-1 Zeros of transfer function, 39-3 ZODH (zero-order-data-hold) sampling, 7-1