Maximum *a posteriori* (MAP): Suppose that the parameter vector $\underline{a}$ is random and the *a posteriori* probability density of $\underline{a}$ given $\underline{y}$, $p_{\underline{A}|\underline{Y}}(\underline{a}|\underline{y})$, is known. In MAP estimation, the parameter vector is selected to maximize $p_{\underline{A}|\underline{Y}}(\underline{a}|\underline{y})$ over the space of parameter vectors $\underline{A}$. When the *a priori* probability density $p_{\underline{A}}(\underline{a})$ is flat over the range of $\underline{a}$, ML and MAP yield the same parameter estimation.

Minimum-Mean-Squared Error (MMSE): Suppose again that the parameter vector $\underline{a}$ is random and the *a posteriori* probability density of $\underline{a}$ given $\underline{y}$, $p_{\underline{A}|\underline{Y}}(\underline{a}|\underline{y})$, is known. In MMSE estimation, the parameter vector is selected by minimizing the mean-squared error $E[(\underline{\hat{a}} - \underline{a})^2]$ which can be shown to result in the conditional *a posteriori* mean $E[\underline{a}|\underline{y}]$; thus, when the maximum of $p_{\underline{A}|\underline{Y}}(\underline{a}|\underline{y})$ equals its mean, the MAP and MMSE estimates are equal.

EXERCISES

13.1 Consider a signal $y[n]$ of the form $y[n] = x[n] + b[n]$ where $x[n]$ is a sinewave, i.e., $x[n] = A\cos(\omega_o n)$, and $b[n]$ is a white noise background disturbance with variance σ^2. In this problem, you investigate the signal properties used in Example 13.1 for $y[n]$, analyzed by a short-time window $w[n]$ of length N_w.

(a) Show that when the sinewave frequency ω_o is larger than the width of the main lobe of the analysis window, it follows that

$$|X(pL, \omega_o)| \approx \left|\frac{A}{2}W(0)\right|$$

where $W(0) = \sum_{n=-\infty}^{\infty} w[n]$ and thus

$$E[|X(pL, \omega_o)|^2] \approx \frac{A^2}{4}\left|\sum_{n=-\infty}^{\infty} w[n]\right|^2$$

where E denotes the expectation operator.

(b) Show that the average power of the windowed noise is constant with frequency, i.e.,

$$E[|B(pL, \omega)|^2] = \sigma^2 \sum_{n=-\infty}^{\infty} w^2[n].$$

(c) Using the property that $x[n]$ and $b[n]$ are uncorrelated, show that the SNR ratio in Equation (13.9) follows. Argue that Δ_w in Equation (13.9) represents the 3-dB bandwidth of the window main lobe.

13.2 For the signal $y[n] = x[n] + b[n]$ with the object random process $x[n]$ uncorrelated with the background noise random process $b[n]$, derive the Wiener filter in Equation (13.10). Then show its time-varying counterpart in Equation (13.11) in terms of signal-to-noise ratio $R(n, \omega) = \hat{S}_x(n, \omega)/\hat{S}_b(\omega)$.

13.3 Consider a filter bank $h_k[n] = w[n]e^{j\frac{2\pi}{N}kn}$ that meets the FBS constraint. In this problem, you develop a single noise suppression filter, applied to all N channel outputs for a noisy input $y[n] = x[n] + b[n]$. Assume the object random process $x[n]$ uncorrelated with the background noise