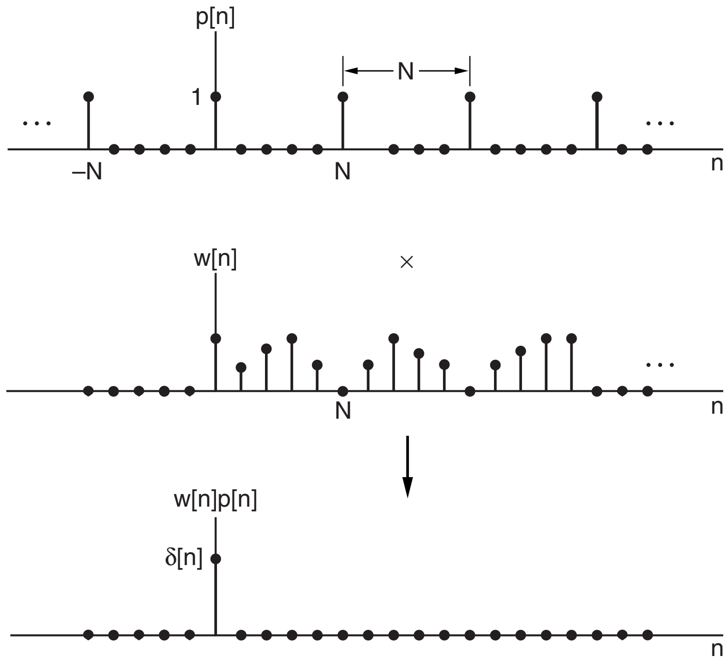

Figure 7.11 Example of an analysis window and how it satisfies the FBS constraint. The analysis-window length is longer than the frequency sampling factor. The sequence $p[n] = \sum_{r=-\infty}^{\infty} \delta[n, n-r]$.

SOURCE: S.H. Nawab and T.F. Quatieri, "Short-Time Fourier Transform" [13]. ©1987, Pearson Education, Inc. Used by permission.

method provided the length of the window is less than the frequency sampling factor N. We can even have $N_w > N$, provided $w[n]$ is chosen such that every Nth sample is zero, i.e.,

$$w[rN] = 0 \quad \text{for } r = -1, 1, -2, 2, -3, 3, \cdots \tag{7.12}$$

as illustrated in Figure 7.11.

Equation (7.12) is known as the *FBS constraint* because this is the requirement on the analysis window that ensures exact signal synthesis with the FBS method. This constraint is more commonly expressed in the frequency domain. Taking the Fourier transform of both sides of Equation (7.11), we obtain (Exercise 7.3)

$$\sum_{k=0}^{N-1} W\left(\omega - \frac{2\pi}{N}k\right) = Nw[0]. \tag{7.13}$$

This constraint essentially states that the frequency responses of the analysis filters should sum to a constant across the entire bandwidth. We have already seen that any finite-length analysis window whose length is less than or equal to the frequency sampling factor N satisfies this constraint. We conclude that a filter bank with N filters, based on an analysis filter of a length less than or equal to N, is *always* an all-pass system. This is not surprising because, from