Some special sequences serve as the building blocks for a general class of discrete-time signals [9]. The unit sample or "impulse" is denoted by

$$\begin{aligned} \delta[n] &= 1, \qquad n = 0 \\ &= 0, \qquad n \neq 0. \end{aligned}$$

The unit step is given by

$$\begin{aligned} u[n] &= 1, \qquad n \geq 0 \\ &= 0, \qquad n < 0 \end{aligned}$$

and can be obtained by summing the unit sample: $u[n] = \sum_{k=-\infty}^{n} \delta[k]$. Likewise, the unit sample can be obtained by differencing the unit step with itself shifted one sample to the right, i.e., forming the first backward difference: $\delta[n] = u[n] - u[n-1]$. The exponential sequence is given by

$$x[n] = A\alpha^n$$

where if A and α are real, then $x[n]$ is real. Moreover, if $0 \leq \alpha < 1$ and $A > 0$, then the sequence $x[n]$ is positive and decreasing with increasing n. If $-1 < \alpha \leq 0$, then the sequence values alternate in sign. The sinusoidal sequence is given by

$$x[n] = A\cos(\omega n + \phi)$$

with frequency ω, amplitude A, and phase offset ϕ. Observe that the discrete-time sinusoidal signal is periodic[1] in the time variable n with period N only if $N = \text{integer} = 2\pi k/\omega$.

The complex exponential sequence with complex gain $A = |A|e^{j\phi}$ is written as

$$\begin{aligned} x[n] &= Ae^{j\omega n} \\ &= |A|e^{j\phi}e^{j\omega n} \\ &= |A|\cos(\omega n + \phi) + j|A|\sin(\omega n + \phi). \end{aligned}$$

An interesting property, which is a consequence of being discrete, is that the complex exponential sequence is periodic in the frequency variable ω with period 2π, i.e., $Ae^{j(\omega+2\pi)n} = Ae^{j\omega n}$. This periodicity in ω also holds for the sinusoidal sequence. Therefore, in discrete time we need to consider frequencies only in the range $0 \leq \omega < 2\pi$. The complex exponential and the above four real sequences serve as building blocks to discrete-time speech signals throughout the text.

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[1] This is in contrast to its continuous-time counterpart $x_a(t) = A\cos(\Omega t + \phi)$ that is always periodic with period $= 2\pi/\Omega$. Here the uppercase frequency variable Ω is used for continuous time rather than the lower case ω for discrete time. This notation will be used throughout the text.