Solution

The MMSE estimate may be found using the following method. Let p_i be the received power at a distance d_i and let $\hat{p}_i$ be the estimate for p_i using the $(d/d_0)^n$ path loss model of Equation (4.67). The sum of squared errors between the measured and estimated values is given by

$$J(n) = \sum_{i=1}^{k} (p_i - \hat{p}_i)^2$$

The value of n which minimizes the mean square error can be obtained by equating the derivative of $J(n)$ to zero, and then solving for n.

(a) Using Equation (4.68), we find $\hat{p}_i = p_i(d_0) - 10n\log(d_i \S 100 \text{ m})$. Recognizing that $P(d_0) = 0$ dBm, we find the following estimates for $\hat{p}_i$ in dBm:

$$\hat{p}_1 = 0,\ \hat{p}_2 = -3n,\ \hat{p}_3 = -10n,\ \hat{p}_4 = -14.77n.$$

The sum of squared errors is then given by

$$\begin{aligned} J(n) &= (0-0)^2 + (-20-(-3n))^2 + (-35-(-10n))^2 + (-70-(-14.77n))^2 \\ &= 6525 - 2887.8n + 327.153n^2 \end{aligned}$$

$$\frac{dJ(n)}{dn} = 654.306n - 2887.8.$$

Setting this equal to zero, the value of n is obtained as $n = 4.4$.

(b) The sample variance $\sigma^2 = J(n)/4$ at $n = 4.4$ can be obtained as follows.

$$\begin{aligned} J(n) &= (0+0) + (-20+13.2)^2 + (-35+44)^2 + (-70+64.988)^2 \\ &= 152.36. \end{aligned}$$

$$\sigma^2 = 152.36/4 = 38.09\,\text{dB}^2$$

therefore

$\sigma = 6.17$ dB, which is a biased estimate. In general, a greater number of measurements are needed to reduce σ^2.

(c) The estimate of the received power at $d = 2$ km is given by

$$\hat{p}(d = 2 \text{ km}) = 0 - 10(4.4)\log(2000/100) = -57.24 \text{ dBm}.$$

A Gaussian random variable having zero mean and $\sigma = 6.17$ dB could be added to this value to simulate random shadowing effects at $d = 2$ km.

(d) The probability that the received signal level will be greater than –60 dBm is given by

$$Pr[P_r(d) > -60 \text{ dBm}] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right) = Q\left(\frac{-60 + 57.24}{6.17}\right) = 67.4\ \%.$$

(e) If 67.4% of the users on the boundary receive signals greater than –60 dBm, then Equation (4.78) or Figure 4.18 may be used to determine that 88% of the cell area receives coverage above –60 dBm.