

# Basic Definitions and DC Circuits

2

This chapter's main objective is to highlight some of the commonly used definitions and fundamental concepts in electric circuits, which are supported by a set of custom-written VIs. These VIs enable students to examine various scenarios in circuits or control panels and, hence, provide an excellent tool for interactive studying. For example, a circuit can be modified easily by varying its controls on the front panel—a series resistance can be zeroed and a parallel resistance can be set to a very high value to introduce a short circuit and an open circuit, respectively. Or a dc offset can be introduced to a programmed waveform to obtain a desired average or root mean square (rms) value, which is supported by the visual display of the waveform.

This chapter is divided into five sections with accompanying custom-written VIs. The first two sections offer some basic explanations about common electrical waveforms and their distinguishing features. We then develop the concept further by studying harmonics in nonsinusoidal waveforms.

Section 2.3, DC Circuits, covers basic circuit topologies and mesh analysis. Section 2.4 presents Thevenin's and Norton's equivalent circuits. In addition, each subsection includes a set of self-study questions that are structured to assist learning and to encourage students to investigate alternative settings on the VIs.

**Educational Objectives** After completing this chapter, students should be able to

- understand the basic concepts in dc circuits, periodic waveforms, and harmonics.
- state the meaning of the terms *periodic* and *rectified waveforms; average, rms, and maximum values; and equivalent resistance*.
- solve for unknown quantities of resistance, current, voltage, and power in series, parallel, and combination circuits.
- examine the concepts of open and short circuits and describe their effects on dc circuits.
- understand Thevenin's and Norton's equivalent circuits.
- create various scenarios with the provided circuits, and verify the results analytically.
- gain skills in virtual instrumentation to create more complex and alternative systems by analyzing the programming block diagrams.

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## 2.1 Periodic Waveforms, and Average and RMS Values

The electric power used for most industrial and household applications is generated and transmitted in the form of a fixed frequency (either 50 Hz or 60 Hz) sinusoidal voltage or current. These signals generated by alternators are time-dependent periodic signals that satisfy the equation

$$x(t) = x(t + nT) \quad (2.1)$$

where  $t$  is the time,  $T$  is the period of  $x(t)$ , and  $n$  is an integer.

Such signals are usually expressed as a perfect sine wave and known as ac quantities. Hence, a representation of an arbitrary ac sinusoidal voltage signal is given as

$$v(t) = V_m \sin \omega t \quad (2.2)$$

where  $V_m$  is the amplitude, and  $\omega$  is the angular frequency in radians/s, which is equal to  $2\pi f$ . The frequency of a periodic signal  $f$  refers to the number of times the signal is repeated in a given time. The period is the time it takes for one cycle to be repeated. The frequency  $f$  and the period  $T$  are reciprocals of each other. (Note that one can represent a sine wave in terms of a cosine wave simply by introducing a phase shift of  $\pi/2$  radians.)

Furthermore, there are other periodic waveforms observed in electrical circuits that can be approximated by time-varying ideal signals. Such approximation is usually done using exponential, linear, or logarithmic functions.

In the real world, however, two definitions for voltage and current waveforms are used to quantify the strength of a time-dependent electrical signal: the average (mean or dc) value and the root mean square (rms or effective) value.

The average value of a voltage signal corresponds to integrating the signal waveform over a period of time, which is given for the voltage signal by

$$V_{\text{ave}} = \frac{1}{T} \int_0^T v(t) dt \quad (2.3)$$

The average value of a time-varying waveform may be considered as the dc voltage equivalent of a battery, which does not vary with time and will be used as a voltage source in dc circuits.

The root mean square value of a signal takes into account the fluctuations of the signal about its average value and is defined for the voltage signal as

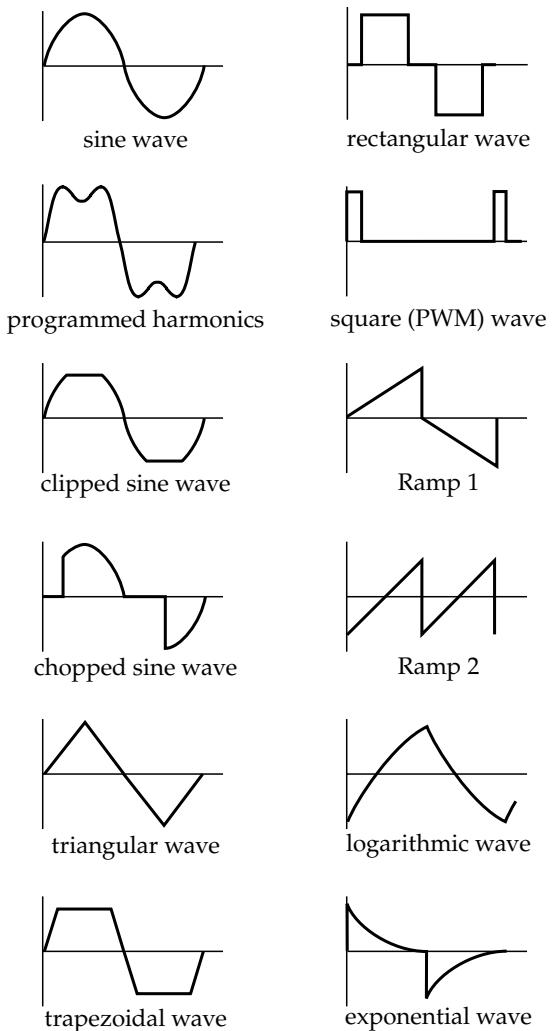
$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} \quad (2.4)$$

For an ideal sinusoidal voltage waveform  $V_{\text{rms}} = V_m / \sqrt{2}$ .

**Note:** True rms meters should be utilized to measure the rms value of any nonsinusoidal waveform. A custom-written LabVIEW VI equipped with a DAQ system can also provide a true rms measurement.

### 2.1.1 Virtual Instrument Panel

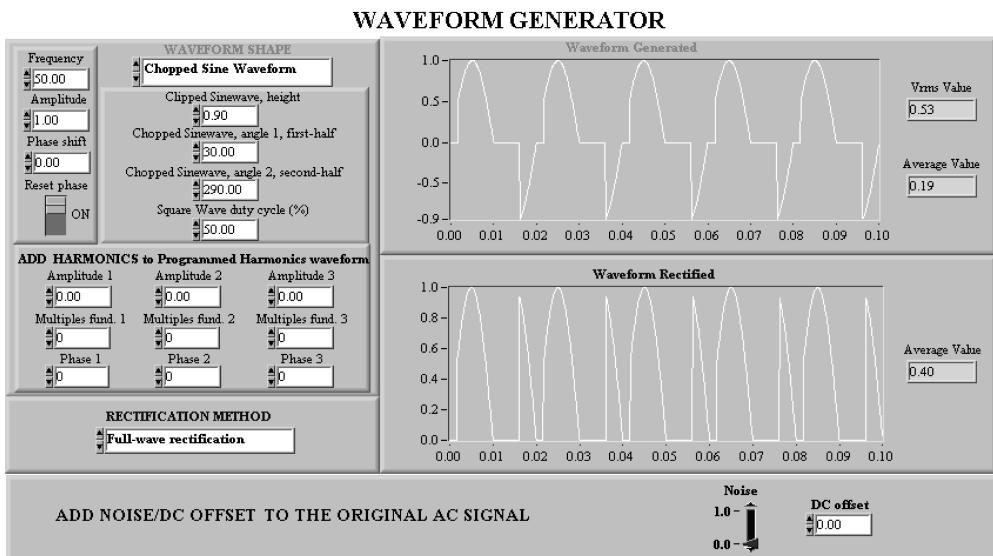
The custom-written VI for this section is named `Waveform Generator.vi` and is located on the accompanying CD-ROM. The objective of the VI is to study the concepts and definitions just introduced using a comprehensive waveform generator that can generate twelve different periodic waveforms (Fig. 2-1): sine wave, programmed harmonics, clipped sine wave, chopped sine wave, triangular wave, trapezoidal wave, rectangular wave, square wave, two different ramp waves, logarithmic wave, and exponential wave. These cover the majority of the practical waveforms featured in electrical and electronic engineering courses.

**Figure 2-1**

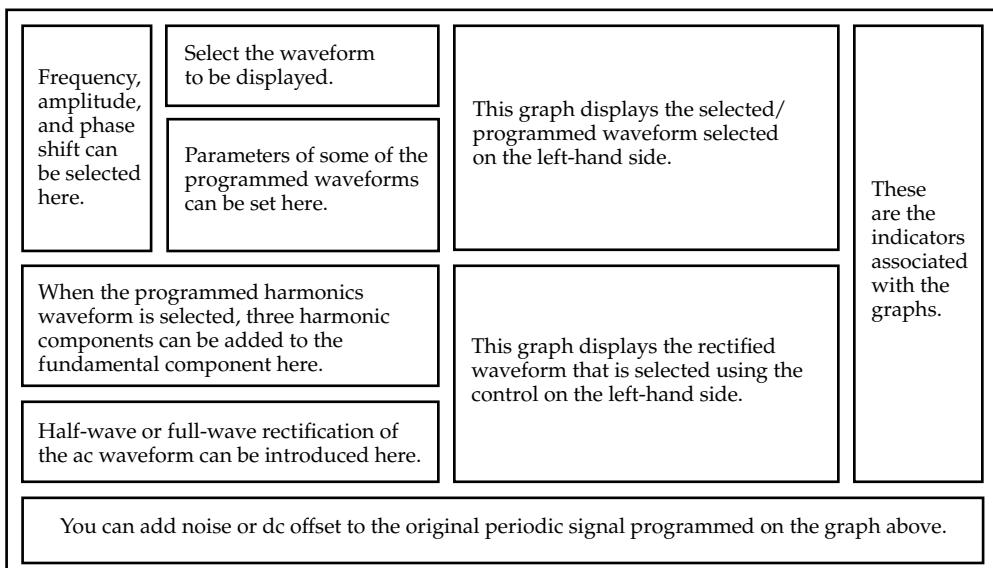
The waveforms that can be generated by the Waveform Generator.vi.

It should be noted here that at least three cycles of a waveform must be contained in the time-domain record for a valid estimate of rms and average values provided in the VI.

As seen on the front panel of the VI (Fig. 2-2a), various parameters of the periodic waveforms (such as frequency, phase, amplitude, dc offset, noise, etc.) can be controlled by the user. The VI can generate various outputs such as waveform graphs and average and rms values of the waveforms that are located next to the associated graph area. Furthermore, some subcontrols are



(a)



(b)

**Figure 2-2**

(a) Front panel and (b) brief user guide of Waveform Generator.vi.

provided to set additional parameters for the specific waveforms, such as duty cycle, chopping angle, clipped angle, and so on. The brief user guide in Fig. 2-2b explains the various features of the front panel.

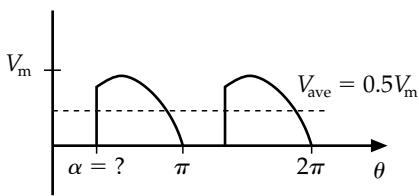
In the subpanel named “Add Harmonics to Programmed Harmonic Waveform,” the user can add three harmonic components onto a fundamental sine wave whose frequencies can be entered as multiples of the fundamental frequency. In addition, phase shifts can be introduced to each harmonic component if desired.

As it is implemented in the VI, practically any waveform can be generated using Formula Node in LabVIEW. I suggest you refer to the block diagram of the VI for the implementation details of the waveforms. After obtaining a basic understanding of LabVIEW programming, I encourage you to use this VI as a starting tool to develop more complex waveforms. In addition, remember that a complex-looking waveform may easily be obtained using a combination of two or three of the waveforms provided.

## 2.1.2 Self-Study Questions

Open and run the VI named `Waveform Generator.vi` in the Chapter 2 `VIs` folder on the accompanying CD-ROM, and investigate the following questions. Remember that the degree of difficulty varies in the following questions. You should verify your findings analytically.

1. Study the rms and average values (both for half-wave and full-wave rectifications) of the waveforms provided in the VI using the default values.
2. Investigate the effect of varying the amplitude, frequency, and phase shift of the waveforms on the calculated rms and average values. Verify that the periods of two waveforms of your choice displayed on the graph are correct. Remember that the period is  $T = 1/f$ .
3. Introduce some dc offset on a periodic waveform of your choice, and observe the change of the full-wave rectified waveform when the dc offset is introduced.
4. Verify that the rms value of a function  $v = 50 + 30 \sin \omega t$  is 54.3 V.
5. Determine a chopped sine wave angle  $\alpha$  that satisfies  $V_{\text{ave}} = 0.5 V_m$  for the waveform given in Fig. 2-3.
6. For the waveforms given in Table 2-1, verify that the values are correct.
7. Select the waveform option Programmed Harmonics, and introduce third, fourth, and fifth harmonics (one at a time). Vary their amplitudes

**Figure 2-3**

Example of a chopped sine wave for question 5.

**Table 2-1** Some selected waveforms and their average values.

Waveform	Average
	$0.32V_m$
	$0.64V_m$
	25
	1
	3.33
	$0.54V_m$
	10
	27.2

gradually and observe the effect of the harmonics on the original pure sine wave.

## 2.2 Periodic Waveforms and Harmonics

In practical electric circuits, voltage and current signals are not pure sine waves. Due to the nonideal behavior of electrical circuits, these signals are usually distorted.

The distortion of the signals in ac circuits can be due to various reasons, such as nonlinear loads (electric arc furnaces, etc.), magnetic saturation in the cores of transformers, or equipment containing switching devices or power supplies. Specifically, due to the switching action in adjustable speed motor drive systems, both the voltage and the current waveforms are highly distorted.

**Table 2-2** Classification of 50 Hz supply harmonics.

Name	Funda-mental	2nd	3rd	4th	5th	6th	7th	Etc.
Frequency (Hz)	50	100	150	200	250	300	350	...

The distortion of dc signals, however, is mainly due to the rectification process. Rectification from an ac source involves various electronic converter circuits and supply transformers that generate ripples.

Nonsinusoidal, distorted waveforms (as illustrated in Table 2-1) can be represented by a series of harmonic components. Each harmonic has a name and frequency (see Table 2-2).

A special case in ac systems occurs when the positive and negative parts of the waveform have negative symmetry, that is,  $f(t) = -f(t + T/2)$ , where  $T$  is the period of the waveform. Hence, there is no dc component, and even harmonics (2nd, 4th, 6th, etc.) will not be generated.

It should be noted that in three-phase ac systems the harmonics are also defined with reference to their sequence, which refers to the direction of rotation with respect to the fundamental. For example, in an induction motor, a positive sequence harmonic generates a magnetic field that rotates in the same direction as the fundamental, while a negative sequence harmonic rotates in the reverse direction. Negative sequence voltages can produce large rotor currents, which may cause the motor to overheat. Zero sequence harmonics are known as Triplens (3rd, 9th, etc.), and they do not rotate but add in the neutral line of the three-phase four-wire system.

In ac circuits, fundamental power (which is produced by fundamental voltage and fundamental current) produces the useful power. The product of a harmonic voltage and the corresponding harmonic current produces a harmonic power. This is usually dissipated as heat in the ac circuits, and, consequently, no useful work is done.

Furthermore, harmonics can cause many other undesirable effects in electric motors, such as torque ripple, noise, vibration, reduction of insulation life, presence of bearing currents, and so on.

Waveforms with discontinuities, such as the ramp and square wave, often have high harmonics, which have amplitudes of significant value compared with the fundamental component. This can be visualized in the VI provided in this section.

The principal solution to reduce or eliminate the harmonics is to add harmonic filters at the source of the harmonics or to use various other techniques, such as programmed switching in motor control applications.

Although the level of distortion in a waveform can be seen by observing the real waveform, the distortion of the signals can be traced to the harmonics it contains using harmonic analysis techniques, one of which will be covered in this section. The tool presented here should provide an insight into the harmonics and enable you to take preventative measures to avoid distortion.

## 2.2.1 Virtual Instrument Panel

A number of periodic waveforms typically encountered in the study of electrical circuits are simulated in the virtual instrument provided in Section 2.1.1. This section develops the concept further and integrates the Waveform Generator.vi and the harmonic analysis module, providing a flexible user interface. In this section, we can decompose a given periodic wave into its fundamental and harmonic components.

The output of the Waveform Generator.vi is applied to the Waveform and Harmonic Analyser.vi (Fig. 2-4a) either as an ac signal or as a dc signal after rectification (half-wave or full-wave). The switch named AC Input or DC Input can be used to achieve the selection.

The well-known Fourier series expresses the periodic wave that is analyzed. Hence the original signal can be reconstructed using a number of terms of the trigonometric series, including the fundamental component of the signal. With more terms included in this reconstruction, the result more nearly resembles the original signal.

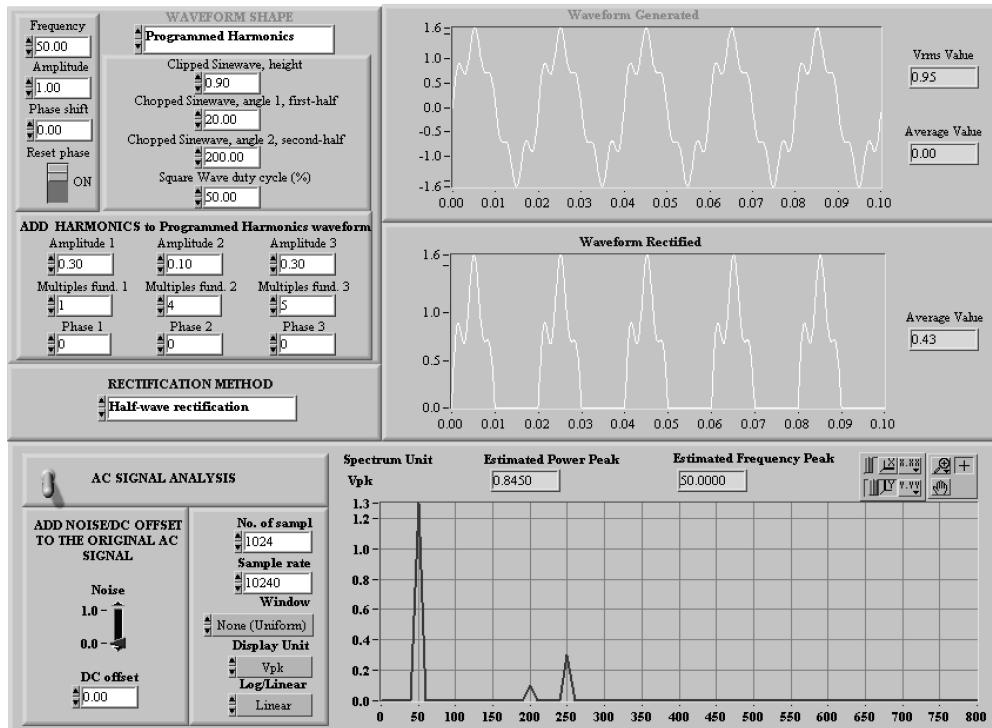
## 2.2.2 Self-Study Questions

Open and run the custom-written VI named Harmonics.vi in the Chapter 2 VIs folder, and investigate the following questions.

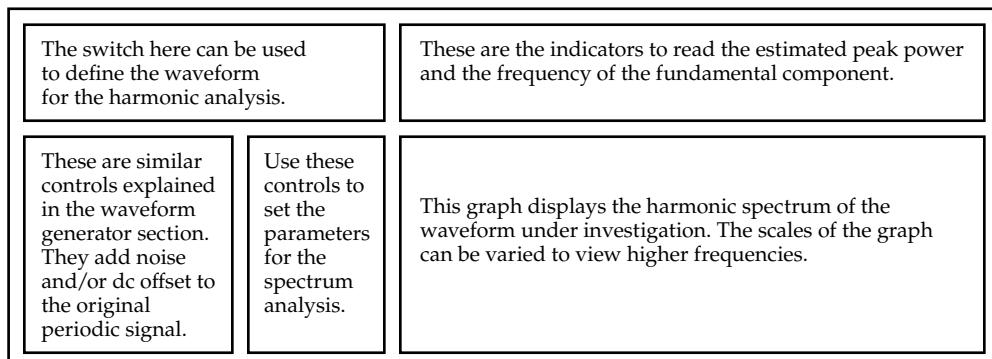
*Note:* When studying a specific case, unless otherwise stated, leave all the control values on the harmonic spectrum analysis panel in their default settings.

1. Certain functions contain a constant term, a fundamental, and a third harmonic. From the given signals available in the Waveform Generator.vi, list the signals, which have these features in their harmonic spectrum.
2. Demonstrate that an ac square wave with an amplitude of 100 V and a frequency of 50 Hz has the following harmonic contents:

### PERIODIC WAVEFORMS and HARMONIC ANALYSIS



(a)



(b)

**Figure 2-4**

(a) Front panel of the complete VI, Waveform and Harmonic Analyser.vi and  
 (b) brief user guide for the additional harmonics front panel.

Harmonic:	Fund.	3rd	5th	7th	9th
Frequency (Hz):	50	150	250	350	450
Amplitude (V):	127.3	42.4	25.5	18.5	14.1

*Hint:* An ac square wave can be obtained by introducing a dc offset to the square waveform with 50% duty cycle.

3. Select a ramp waveform and find the trigonometric Fourier series using the first three harmonic components displayed on the harmonic spectrum graph.
4. Waveform synthesis is a combination of the harmonics so as to form the actual waveform. Demonstrate that the ramp generated in question 3, which utilized three harmonic components, is not sufficient to form the actual waveform. Propose a solution.
5. The output of a full-wave rectified sine wave consists of a series of harmonics. Demonstrate that the Fourier representation of such a periodic wave is

$$f(t) = \frac{2V_m}{\pi} \left( 1 - \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t - \frac{2}{35} \cos 6\omega t - \dots \right)$$

6. Select a clipped sine wave with a clipped height of  $0.2V_m$ , and compare its harmonic contents to a trapezoidal waveform with an amplitude of 0.2.
7. Demonstrate that the average value of a waveform displayed in the corresponding indicator is equal to the magnitude of the dc component observed on the harmonic analysis graph.

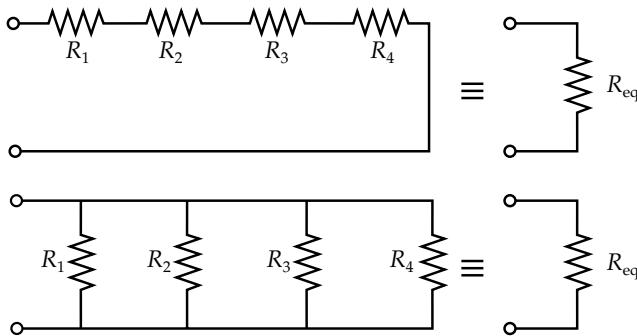
## 2.3 DC Circuits

### 2.3.1 Equivalent Resistance and Series/Parallel Resistance Circuits

The basic circuit element we will use in this section is an ideal resistor,  $R$ . The current in an ideal resistor is linearly related to the voltage across it, and it has a value, which is time-invariant.

$$V = iR \quad (2.5)$$

Resistors can be connected in series or in parallel in electric circuits. When resistors are connected in series, they share the same current, and the voltages

**Figure 2-5**

Series and parallel resistance circuit.

across them add to give the total voltage. The opposite is true in parallel resistance circuits; that is, parallel components share the same voltage, and their currents add to give the total current.

The equivalent resistances of a series and a parallel circuit (Fig. 2-5) can be calculated using the following formulas. These illustrate the case involving four elements.

$$R_{eq(series)} = R_1 + R_2 + R_3 + R_4 \quad (2.6)$$

$$R_{eq(parallel)} = \frac{1}{(1/R_1 + 1/R_2 + 1/R_3 + 1/R_4)} \quad (2.7)$$

Furthermore, the three basic circuits given in Fig. 2-6 will be used to study voltage and current division in the resistance circuits.

In Fig. 2-6a (voltage divider circuit), two resistors are connected in series across a voltage source. As seen in the figure, the resistors share the same current, and the voltages across them are proportional to their resistances. In addition, the power dissipated in each resistor can be calculated as follows.

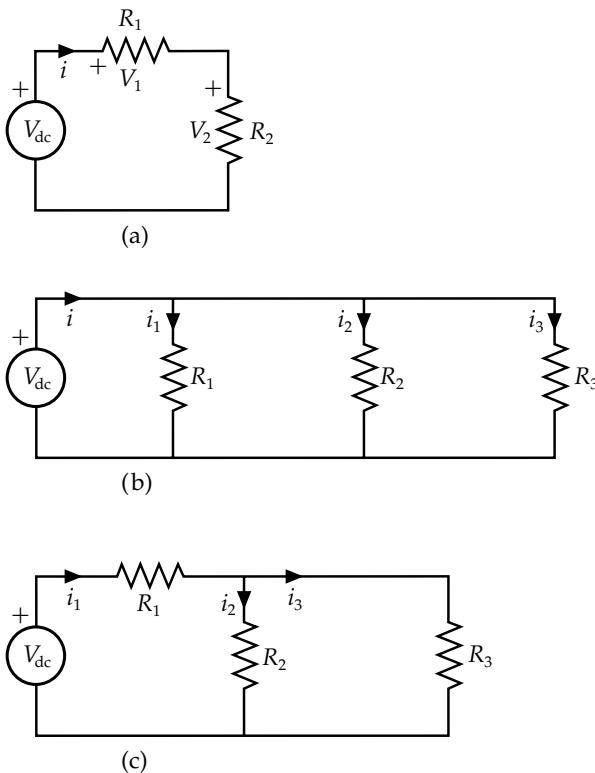
$$i = \frac{V_{dc}}{(R_1 + R_2)} \quad (2.8)$$

$$V_2 = R_2 i = V_{dc} \frac{R_2}{(R_1 + R_2)} \quad (2.9)$$

$$V_1 = R_1 i = V_{dc} \frac{R_1}{(R_1 + R_2)} \quad (2.10)$$

$$P_{R1} = V_1 i = i^2 R_1 \quad (2.11)$$

$$P_{R2} = V_2 i = i^2 R_2 \quad (2.12)$$

**Figure 2-6**

Voltage and current division circuits: (a) voltage divider circuit (series resistance circuit), (b) current divider circuit (parallel resistance circuit), and (c) series/parallel circuit.

The current divider circuit is studied using the circuit given in Fig. 2-6b. Since the resistors are in parallel, they share the same voltage. The current division between the resistors is inversely proportional to their resistances or directly proportional to their conductance,  $G$ . Note that the conductance is the reciprocal of resistance ( $G = 1/R$ ).

$$i = i_1 + i_2 + i_3 = V_{dc} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = V_{dc}(G_1 + G_2 + G_3) \quad (2.13)$$

$$i_1 = V_{dc} \frac{1}{R_1} = i \frac{(1/R_1)}{(1/R_1 + 1/R_2 + 1/R_3)} = i \frac{G_1}{(G_1 + G_2 + G_3)} \quad (2.14)$$

$$i_2 = V_{dc} \frac{1}{R_2} = i \frac{(1/R_2)}{(1/R_1 + 1/R_2 + 1/R_3)} = i \frac{G_2}{(G_1 + G_2 + G_3)} \quad (2.15)$$

$$i_3 = V_{dc} \frac{1}{R_3} = i \frac{(1/R_3)}{(1/R_1 + 1/R_2 + 1/R_3)} = i \frac{G_3}{(G_1 + G_2 + G_3)} \quad (2.16)$$

A combination circuit is given in Fig. 2-6c. As seen in the figure, two parallel resistors are connected in series with a single resistor. Therefore, an equivalent circuit can be obtained that is similar to the circuit studied in Fig. 2-6a. Hence, both voltage and current divider rules can be applied to the original circuit, where

$$i_1 = i_2 + i_3 \quad (2.17)$$

$$i_1 = \frac{V_{dc}}{(R_1 + R_{eq})}, \quad R_{eq} = \frac{1}{(1/R_1 + 1/R_2)} \quad (2.18)$$

$$V_{eq} = R_{eq}i_1 = V_{dc} \frac{R_{eq}}{(R_1 + R_{eq})} \quad (2.19)$$

$$i_3 = i_1 \frac{(1/R_3)}{(1/R_2 + 1/R_3)} = i_1 \frac{G_3}{(G_2 + G_3)} = \frac{V_{eq}}{R_3} \quad (2.20)$$

### 2.3.1.1 Virtual Instrument Panel

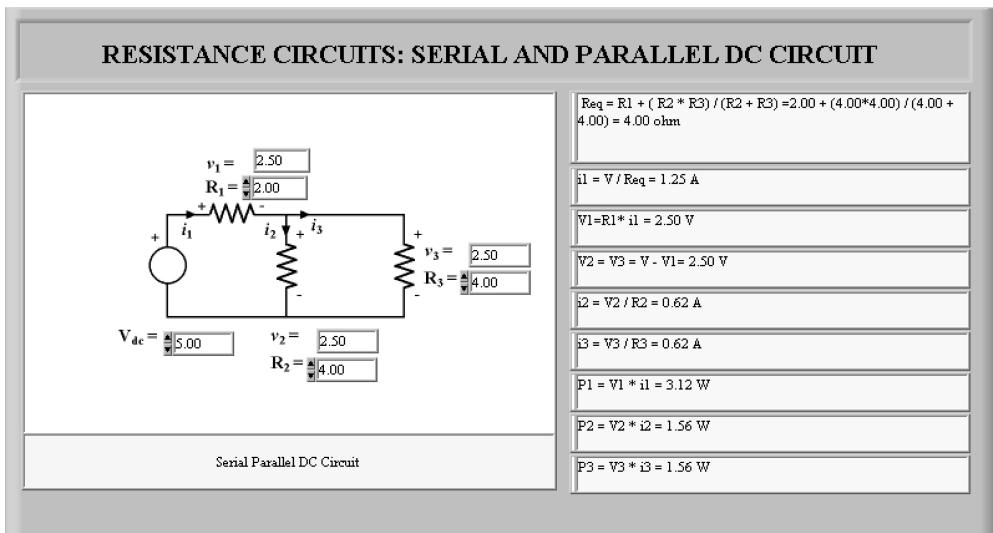
The LabVIEW VI implemented in this section (Fig. 2-7) contains six different circuit options covering the previously discussed circuit topologies, including a circuit study of the concept of mesh analysis, which is discussed in the following section. The desired circuit can be selected from the library file, which contains all the circuits discussed.

The VIs of the circuits here calculate the voltage, current, and power of each circuit element and present them in the same format as given in conventional textbooks. My intention is to emphasize the effect of changing certain circuit parameters on the current and the power of the other circuit elements.

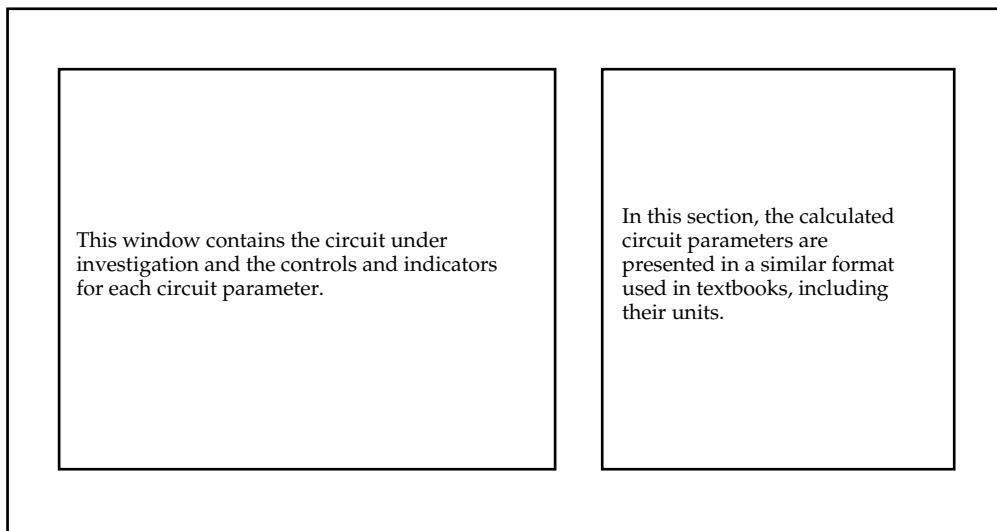
Moreover, you can experiment with open-circuit and short-circuit concepts in any branch of the circuit by varying the circuit parameters depending on their connection.

### 2.3.1.2 Self-Study Questions

Open and run the custom-written VIs located in `Resistance Circuits .11b`, in the Chapter 2 VIs folder. Remember that if a circuit contains more components than the circuits presented here, you can subdivide your circuit into small subsections that are similar to the other circuits analyzed. Furthermore, if the value of a resistance has to be set to  $0 \Omega$ , it means a short circuit of that branch. However, if the value of a resistance is very large (compared with the resistances of the other components), the branch can practically be assumed open circuit.



(a)



(b)

**Figure 2-7**

(a) A sample front panel and (b) brief user guide for the VIs in the Resistance Circuits.llb.

1. Select the options Series Resistance Circuit and Parallel Resistance Circuit, respectively, and vary the values of the resistances in the circuit to estimate the equivalent resistance. First, start with one resistance only, then gradually add more resistances, and verify your results analytically.
2. One common application of the voltage divider circuit is to reduce a high voltage to the low levels that are used in signal conditioning circuits. First, select the Voltage Divider Circuit option from the Menu Ring of the VI. We would like to measure a 200 V voltage that should be scaled down to 5 V to be linked to a computer. Find the values of the resistances to make sure that their powers do not exceed 1 W.
3. Select the Current Divider Circuit, and set the parameters as  $V_{dc} = 12$  V,  $R_1 = 2 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 10,000 \Omega$  (to introduce an open circuit), and estimate the currents in each branch. First double and then halve the values of the resistances and compare your results.
4. Select the Series/Parallel Circuit and set the parameters as  $V_{dc} = 12$  V,  $R_1 = 2 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 2 \Omega$ . Comment on the powers of each component. What is the power taken from the supply  $V_{dc}$ ?
5. Select the Series/Parallel Circuit and set the parameters as  $V_{dc} = 12$  V,  $R_1 = 2 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 10,000 \Omega$ , and record the current values  $I_1$ ,  $I_2$ , and  $I_3$ . Change  $R_3$  to  $20,000 \Omega$ , and find the new values of  $I_1$ ,  $I_2$ , and  $I_3$ . Comment on your findings.

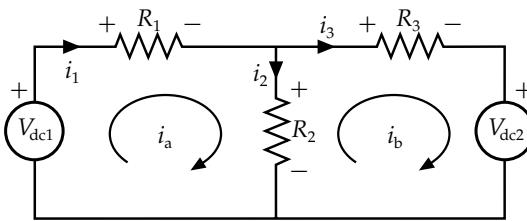
### 2.3.2 Mesh Analysis

In solving electric circuits, Kirchhoff's laws, mesh analysis (unknowns are currents), and nodal analysis (unknowns are voltages) can be utilized. These can provide all the independent current and voltage equations.

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents entering a node (where two or more elements have a common terminal) is equal to zero. In standard notation, the ingoing currents are considered negative, and the outgoing currents are considered positive.

Kirchhoff's Voltage Law (KVL), on the contrary, states that the algebraic sum of voltages around a loop (consisting of nodes and branches, which form simple closed paths) is equal to zero. In standard notation, an arrow or  $+$ / $-$  signs are used to indicate the sign of the voltage potential. The  $+$  sign is equivalent to the head of the arrow, which is an arbitrary choice.

Mesh analysis starts by defining a current circulating around each mesh (the loops corresponding to the open areas in the circuit without any crossovers). The element currents are then the algebraic sums of the mesh currents

**Figure 2-8**

A sample electric circuit used in mesh analysis.

that pass through them. Since each mesh current enters and leaves a node, KCL is automatically satisfied. The resulting equations are the result of KVL applied to each mesh, hence the unknowns are the mesh currents. The remaining unknowns in the circuit (elements' currents and voltages) can be calculated using the mesh currents.

For the simple circuit given in Fig. 2-8, if KVL is written for each mesh using the standard notation in relation to the mesh currents  $i_a$  and  $i_b$ ,

$$-v_{R2} - v_{R1} + v_{dc1} = -R_2(i_a - i_b) - R_1i_a + V_{dc1} = 0 \quad (2.21)$$

$$-v_{dc2} - v_{R3} + v_{R2} = -V_{dc2} - R_3i_b + R_2(i_b - i_a) = 0 \quad (2.22)$$

If equations 2.21 and 2.22 are rearranged, the unknown currents,  $i_a$  and  $i_b$ , can be calculated.

$$V_{dc1} = i_a(R_1 + R_2) - i_bR_2 \quad (2.23)$$

$$V_{dc2} = i_aR_2 - i_b(R_2 + R_3) \quad (2.24)$$

Hence, the currents and voltages of the resistance elements are

$$i_1 = i_a, \quad v_{R1} = R_1i_a \quad (2.25)$$

$$i_2 = i_a - i_b, \quad v_{R2} = R_2(i_a - i_b) \quad (2.26)$$

$$i_3 = i_b, \quad v_{R3} = R_3i_b \quad (2.27)$$

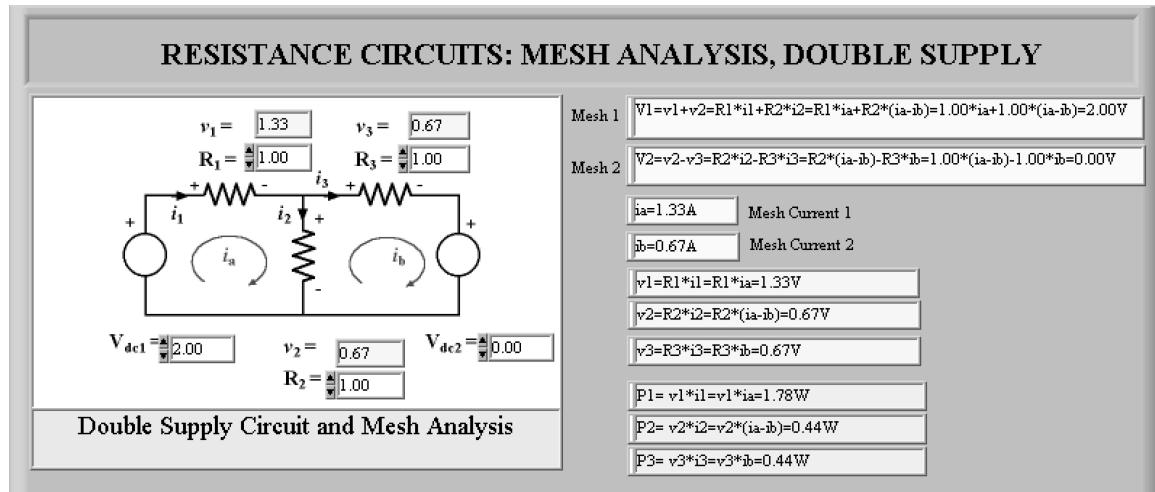
### 2.3.2.1 Virtual Instrument Panel

The front panel of the `Mesh Analysis.vi` illustrated in Fig. 2-9 can be accessed via `Resistance Circuits.llb`.

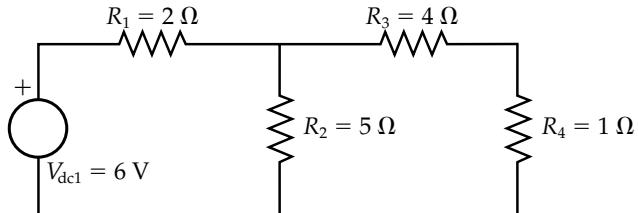
### 2.3.2.2 Self-Study Questions

Open and run the VI named `Resistance Circuits.vi`, in the Chapter 2 VIs folder, and select the option Mesh Analysis.

- Consider the circuit parameters as  $V_{dc1} = 42\text{ V}$ ,  $V_{dc2} = 10\text{ V}$ ,  $R_1 = 6\Omega$ ,  $R_2 = 3\Omega$ ,  $R_3 = 4\Omega$ , and confirm that the mesh currents  $I_a$  and  $I_b$  are  $4.889\text{ A}$  and  $0.667\text{ A}$ , respectively. Verify your findings by manual calculations.

**Figure 2-9**

Front panel of the Mesh Analysis Circuit.vi.

**Figure 2-10**

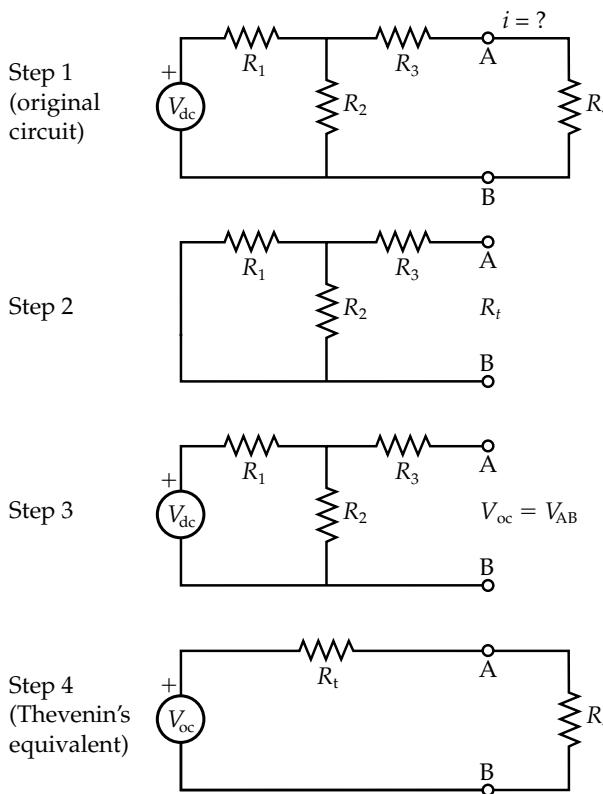
The circuit for question 2, which can be solved by using mesh analysis.

2. Consider the circuit given in Fig. 2-10, and confirm that the voltage across the resistances  $R_2$  and  $R_4$  is 3.333 V and 0.667 V, respectively.

*Hint:* Use an equivalent resistance for  $R_3$  and  $R_4$  in Fig. 2-10, and set  $V_{dc2} = 0$  V in the front panel circuit.

## 2.4 Thevenin's and Norton's Equivalent Circuits

Thevenin's and Norton's equivalent circuits are used to transform complex circuits to simple circuits, voltage sources into current sources, or current sources to voltage sources (which are also known as source transformations), provided that there is an appropriate resistance in series with the voltage source or in parallel with the current source.

**Figure 2-11**

The steps illustrating how to obtain a Thevenin equivalent circuit for resistance circuits containing one independent source only.

Consider a circuit with resistance elements and a voltage source with identified output terminals A and B. A Thevenin's equivalent circuit can be constructed by a series combination of an ideal voltage source  $V_{oc}$  and a resistance  $R_t$ , where  $V_{oc}$  is the open-circuit voltage at the identified terminals and  $R_t$  is the Thevenin's equivalent of the resistor.

The resistor  $R_t$  is the ratio of the open-circuit voltage to the short-circuit current at the terminals A–B. The steps followed to obtain this transformation are visually illustrated in Fig. 2-11, which is also used in the LabVIEW simulation.

As shown in Fig. 2-11, the principal aim is to find the current that flows through the resistor  $R_4$ . This can easily be estimated if the circuit on the left-hand side of the terminals A–B is transformed to a simple circuit given in Step 4 that contains the Thevenin equivalent circuit.

In Step 2, short-circuiting the source terminals deactivates the voltage source and allows the equivalent Thevenin resistance  $R_t$  to be calculated. Note that if a current source is present in a circuit, it should be open-circuited in Step 2.

**Table 2-3** Three methods of finding Thevenin and Norton equivalent circuits.

Features of the Resistance Circuits	Steps to Obtain <b>Thevenin</b> Equivalent Circuit	Steps to Obtain <b>Norton</b> Equivalent Circuit
With independent sources	<ul style="list-style-type: none"> <li>Deactivate the sources and find <math>R_t</math></li> <li>Find open-circuit voltage <math>v_{oc}</math> with the sources included</li> </ul> <p><i>Sample:</i> Fig. 2-12a</p>	<ul style="list-style-type: none"> <li>Deactivate the sources and find <math>R_t</math></li> <li>Find short-circuit current <math>i_{sc}</math> with the sources included</li> </ul> <p><i>Sample:</i> Fig. 2-12d</p>
With independent and dependent sources or With independent sources	<ul style="list-style-type: none"> <li>Find open-circuit voltage <math>v_{oc}</math> with the sources included</li> <li>Find short-circuit current <math>i_{sc}</math> by short circuiting the terminals A and B.</li> <li>Calculate <math>R_t = v_{oc}/i_{sc}</math></li> </ul> <p><i>Sample:</i> Fig. 2-12b</p>	<ul style="list-style-type: none"> <li>Find short-circuit current <math>i_{sc}</math> with the sources included</li> <li>Find open-circuit voltage <math>v_{oc}</math> at the terminals A and B.</li> <li>Calculate <math>R_t = v_{oc}/i_{sc}</math></li> </ul> <p><i>Samples:</i> Fig. 2-12e and 2-12f</p>
With dependent sources where $v_{oc} = 0$ or $i_{oc} = 0$	<ul style="list-style-type: none"> <li>Where <math>v_{oc} = 0</math></li> <li>Connect a 1 A current source to the terminals A and B, and calculate <math>v_{AB}</math>.</li> <li>Estimate <math>R_t = v_{AB}/1 \text{ A}</math></li> </ul> <p><i>Sample:</i> Fig. 2-12c</p>	<ul style="list-style-type: none"> <li>Where <math>i_{sc} = 0</math></li> <li>Connect a 1 A current source to the terminals A and B, and calculate <math>v_{AB}</math>.</li> <li>Estimate <math>R_t = v_{AB}/1 \text{ A}</math></li> </ul>

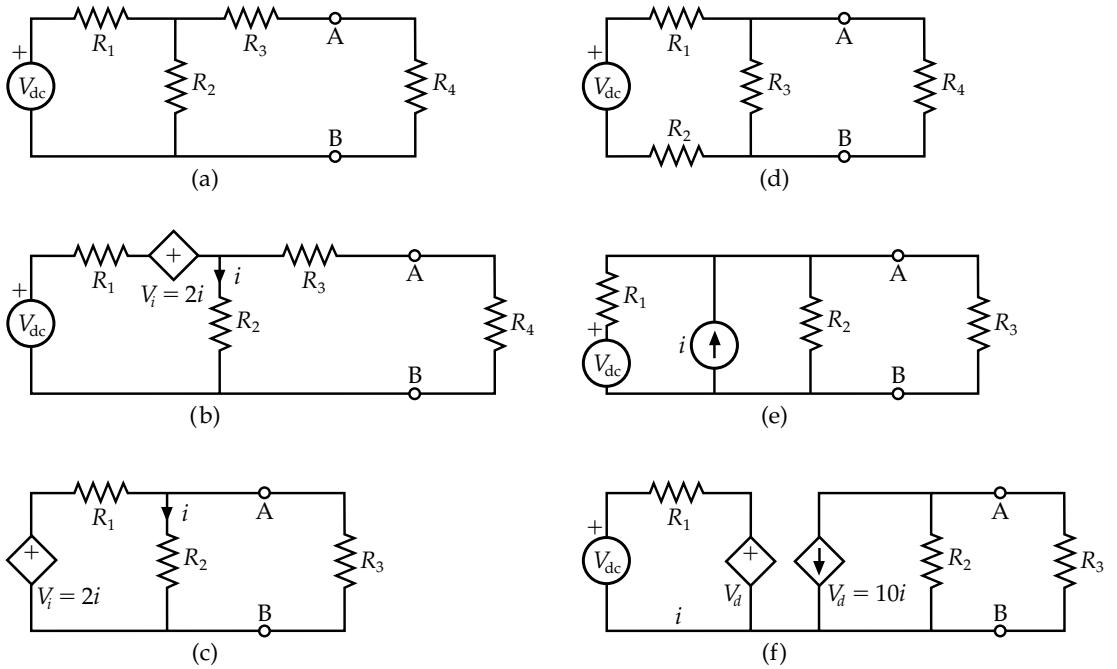
In Step 3, the open-circuit voltage  $v_{oc}$  across the terminals A–B is calculated, and the Thevenin equivalent circuit is replaced with the original circuit in the final step, Step 4.

The Norton's equivalent circuit can also be constructed with a single current source equal to the short-circuit current at terminals A–B, in parallel with a single resistance. The resistance in the Norton equivalent is the same as the Thevenin resistance.

As summarized in Table 2-3, three methods can be identified for the distinct electrical circuits, which can be used to determine Thevenin and Norton equivalent circuits. Six distinct electric circuits illustrated in Fig. 2-12 are used to study Thevenin and Norton equivalent circuits.

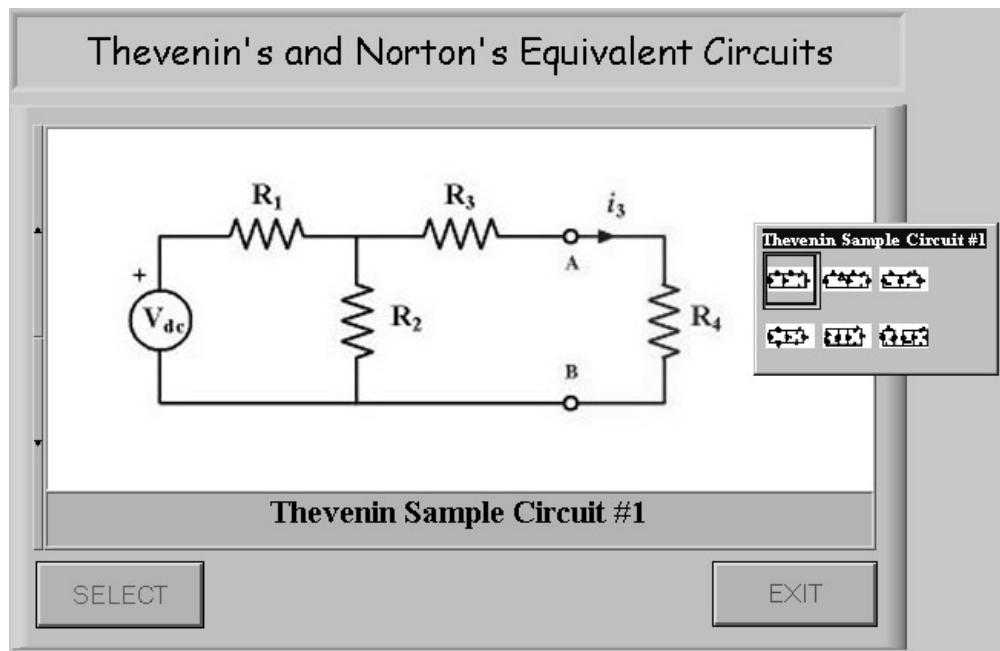
## 2.4.1 Virtual Instrument Panel

The objective of this section is to study Thevenin's and Norton's equivalent circuits in sufficient detail, which requires the custom-written Thevenin Norton.vi. Two front panels given in Fig. 2-13 illustrate the layout of the Thevenin and Norton equivalent circuits with the associated steps.



**Figure 2-12**

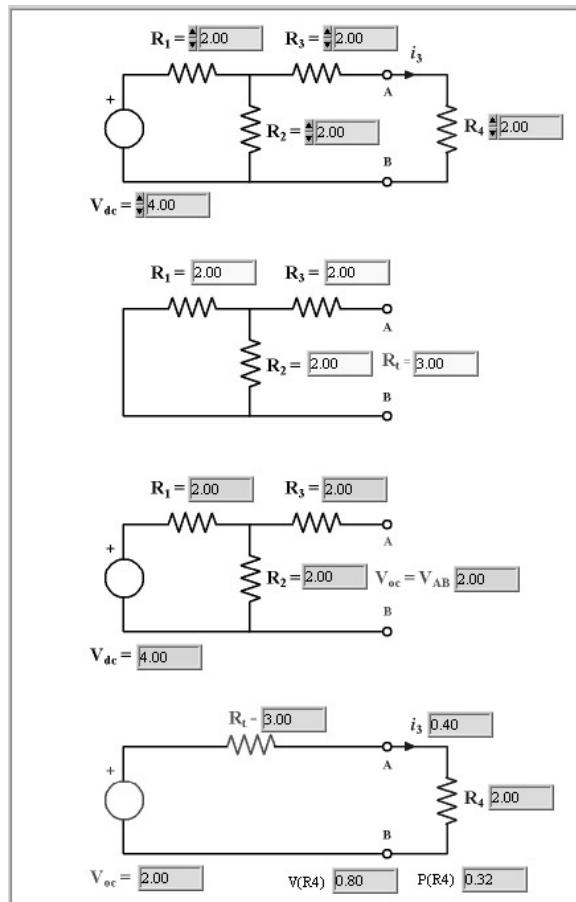
(a), (b), and (c) Typical electric circuits used to obtain Thevenin equivalent circuits. (d), (e), and (f) Typical electric circuits used to obtain Norton equivalent circuits.



(a)

**Figure 2-13**

(a) The main front panel of Thevenin Norton.vi and (b) a sample front panel. (cont.)



(b)

**Figure 2-13**

Continued

## 2.4.2 Self-Study Questions

Open and run the custom-written VI named `Thevenin Norton.vi` in the Chapter 2 VIs folder, and study the following questions.

- Consider the circuit parameters for the circuit given in Part Thevenin (1) as  $V_{dc} = 12 \text{ V}$ ,  $R_1 = 3 \Omega$ ,  $R_2 = 6 \Omega$ ,  $R_3 = 7 \Omega$ ,  $R_4 = 3 \Omega$ . Using the Thevenin's equivalent of the circuit, find the voltage across the output resistor  $R_4$ .

*Answer:  $V_{AB} = 2 \text{ V}$  (for  $V_{oc} = 8 \text{ V}$ ,  $R_t = 9 \Omega$ )*

### Thevenin Sample Circuit #1

#### STEP 1

This is the original circuit with load attached, represented by  $R_4$  and shown in blue. We wish to calculate  $i_3$ , the current being delivered to the load, and hence the power being delivered to the load.

Enter in values for  $V_{dc}$  (the independent voltage source),  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ .

#### STEP 2

To find the equivalent (Thevenin) resistance  $R_t$  of the circuit, all independent sources have been set to zero. Thus,  $V_{dc}$  has been short-circuited.

$$\begin{aligned} R_t &= (R_1 // R_2) + R_3 \\ &= R_1 R_2 / (R_1 + R_2) + R_3 \end{aligned}$$

Note that the circuit becomes purely resistive.

#### STEP 3

To find the open-circuit (Thevenin) voltage of the circuit, we open-circuit its two terminals, A and B, and then calculate the voltage that appears across these two terminals.

$$\text{Here, } V_{oc} = V_{dc} \cdot R_2 / (R_1 + R_2) \quad (\text{Voltage Divider Rule})$$

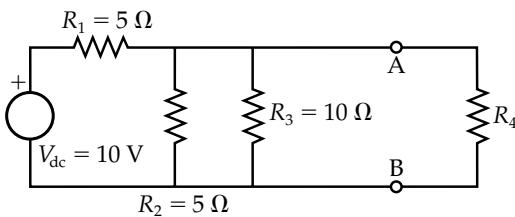
#### STEP 4

This is the final Thevenin equivalent circuit for the 1-port network: an ideal voltage source  $V_{oc}$  in series with an equivalent resistance  $R_t$ .

The current  $i_3$  being delivered to the load can now very easily be found as  $i_3 = V_{oc} / (R_t + R_4)$

The voltage across  $R_4$  is  $V(R_4) = i_3 \cdot R_4$ , and the power being delivered is  $P(R_4) = V(R_4) \cdot i_3 \text{ Watts}$ .

[RETURN](#)

**Figure 2-14**

The circuit diagram for question 5.

- Consider the circuit parameters for the circuit given in Part Thevenin (2) as  $V_{dc} = 20 \text{ V}$ ,  $R_1 = 6 \Omega$ ,  $R_2 = 6 \Omega$ , and  $R_3 = 10 \Omega$ . Determine the Thevenin's equivalent circuit.

*Answer:*  $V_{oc} = 12 \text{ V}$ ,  $R_t = 13.6 \Omega$

- Consider the circuit given in Part Thevenin (3), where  $R_1 = 3 \Omega$  and  $R_2 = 6 \Omega$ . Determine the Thevenin's equivalent circuit.

*Answer:*  $V_{oc} = 0 \text{ V}$ ,  $R_t = 3.85 \Omega$

- Consider the circuit parameters for the circuit given in Part Norton (1) as  $V_{dc} = 15 \text{ V}$ ,  $R_1 = 8 \text{ k}\Omega$ ,  $R_2 = 4 \text{ k}\Omega$ , and  $R_3 = 6 \text{ k}\Omega$ . Determine the Norton's equivalent circuit.

*Answer:*  $i_{sc} = 1.25 \text{ mA}$ ,  $R_n = 4 \text{ k}\Omega$

- Consider the circuit given in Fig. 2-14 and determine the Thevenin's equivalent circuit. *Hint:* You can use the circuit given in Part Thevenin (1) of the custom-written VI. However, you have to include an equivalent resistance into the control box of  $R_2$  and input  $R_3 = 0$ .

*Answer:*  $V_{oc} = 4 \text{ V}$ ,  $R_t = 2 \Omega$

## 2.5 References

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