Example 2.8: Using Steam as a Reference Fluid

Estimate the density of carbon dioxide at 35 bar, 75 °C using data only from the steam tables.

Solution According to the correlation of corresponding states, water and CO_2 have approximately the same compressibility factor at the same reduced state. The conditions given for CO_2 correspond to the reduced coordinates (see Example 2.7):

$$P_r = \frac{35}{73.8} = 0.474, \quad T_r = \frac{75 + 273.15}{304.2} = 1.14.$$

Using the critical constants of water, $P_c = 220.5$ bar, $T_c = 647.3$ K, the corresponding state for water is

$$P = (0.474)(220.5 \text{ bar}) = 104.573 \text{ bar}$$
$$T = (1.14)(647.3 \text{ K}) = 740.82 \text{ K} = 467.67 \text{ °C}.$$

The volume is obtained by interpolation from the steam tables. We find $V = 0.0303815$ m^3/kg, or

$$V = (0.0303815 \text{ m}^3/\text{kg})(18.015 \times 10^{-3} \text{ kg/mol}) = 0.000547323 \text{ m}^3/\text{mol}.$$

The compressibility factor is

$$Z = \frac{PV}{RT} = \frac{(104.573 \text{ bar} \times 10^5 \overset{\text{Pa/bar}}{\cancel{\text{J/m}^3\text{/bar}}})(0.000547323 \text{ m}^3/\text{mol})}{(8.314)(740.82 \text{ K})} = 0.9293.$$

According to the corresponding-states correlation, this is also equal to the compressibility factor of carbon dioxide at 35 bar, 348.15 K. The molar volume of carbon dioxide is

$$V = \frac{ZRT}{P} = \frac{(0.9293)(8.314 \text{ J/mol K})(348.15 \text{ K})}{(35 \text{ bar})} = 7.685 \times 10^{-4} \text{ m}^3/\text{mol}$$

(35 bar) (10^5 Pa/bar)

and the density is

$$\rho = \frac{M_m}{V} = \frac{44 \times 10^{-3} \text{ kg/mol}}{7.685 \times 10^{-4} \text{ m}^3/\text{mol}} = 57.3 \text{ kg/m}^3.$$

Comments The agreement is quite good with the reported value from the NIST tables [1], which is 59.72 kg/m^3. Generally, however, water is not a good reference fluid because of its polarity and hydrogen-bonding ability. In this example the result was satisfactory because the state is vapor. In the gas phase, the chemical character of the molecule is not as important as in liquids because intermolecular distances are large and the effect of molecular interactions is small. Accordingly, very dissimilar molecules behave in very much the same way, as far as volumetric properties are concerned. If the state were liquid, the above calculation would have likely produced a rather poor approximation for the density of carbon dioxide.