

**Example 4.8:** Reversible Adiabatic Compression of Steam

Steam is compressed in a closed system from 1 bar, 200 °C to 20 bar. Calculate the work if the compression is reversible and adiabatic. Repeat the calculation assuming steam to be an ideal gas.

**Solution** (a) *Using the steam tables.* From the steam tables we find  $S_A = 7.8356$  kJ/kg K. The reversible adiabatic process is isentropic; therefore,  $S_B = S_A = 7.8356$  kJ/kg K. In the final state we know pressure (20 bar) and entropy; therefore, the state is fully specified. The temperature and internal energy are obtained by interpolation at  $P_B = 20$  bar,  $S_B = 7.8356$  kJ/kg K and the results are summarized in the table below:

	A	B
$P$ (bar)	1	20
$T$ (°C)	200	652.3
$U$ (kJ/kg)	2658.2	3385.03
$S$ (kJ/kg K)	7.8356	7.8356

The amount of work is

$$W = \Delta U_{AB} = (3385.03 - 2658.2) \text{ (kJ/kg)} = 726.83 \text{ kJ/kg.}$$

(b) *Treating steam as an ideal gas.* Solving eq. 3.44 for the final temperature we obtain

$$T_B = T_A \left( \frac{P_B}{P_A} \right)^{R/\bar{C}_{P\log}^{\text{ig}}} \quad [\text{A}]$$

The ideal-gas heat capacity of steam is

$$C_P^{\text{ig}}/R = 4.4 - 4.19 \times 10^{-3} T + 1.41 \times 10^{-5} T^2 - 1.56 \times 10^{-8} T^3 + 6.32 \times 10^{-12} T^4,$$

with  $T$  in kelvin. Equation [A] must be solved numerically for the final temperature. One way to do this by iteration is as follows: assume  $\bar{C}_{P\log}^{\text{ig}} = C_P^{\text{ig}}(T_A)$  and calculate the final temperature from eq. [A]; using this temperature calculate a new value of  $\bar{C}_{P\log}^{\text{ig}}$  and calculate a new temperature from eq. [A]. This procedure is repeated until temperature does not change appreciably any more. These iterations are summarized below:

Iteration	$\bar{C}_{P\log}^{\text{ig}}/R$	$T_B$ (K)		
1	4.00479	999.694		4.220 962.30
2	4.27126	954.114		4.490 922.10
3	4.25279	957.023	←	4.468 925.03
4	4.25395	956.840		4.470 924.82
5	4.25387	956.852		4.470 924.84

The procedure converges to final temperature  $T_B = \del{956.85} \text{ K} = \del{683.7} \text{ °C}$ . The amount of work is

$$W = \int_{T_A}^{T_B} C_V^{\text{ig}} dT = \int_{T_A}^{T_B} (C_P^{\text{ig}} - R) dT = \int_{T_A}^{T_B} C_P^{\text{ig}} dT - R(T_B - T_A).$$

The integral is calculated using the polynomial expression for the heat capacity and the result is ~~14162.8~~ J/mol, or

$$W = \frac{13145.8 \text{ J/mol}}{18.011 \times 10^{-3} \text{ kg/mol}} \frac{10^{-3} \text{ kJ/J}}{729.7} = 786.2 \text{ kJ/kg.}$$

*Comments* ~~The ideal-gas calculation over estimates the temperature by about 50 K (about 3%) and the work by about 8%.~~

The agreement with the calculation from the steam tables is very good. This is because steam at the conditions of this problem is fairly close to the ideal-gas state.

#### Example 4.9: Using Entropy to Calculate Heat

Steam undergoes reversible, isothermal compression in a closed system from initial state  $T_A = 400^\circ\text{C}$ ,  $P = 24 \text{ bar}$  to  $T_B = 400^\circ\text{C}$ ,  $P_B = 40 \text{ bar}$ . Calculate the amount of work and heat exchanged.

**Solution** In reversible processes, the exact equality  $dS = dQ/T$  holds (see eq. [4.7]) and this can be useful in obtaining the amount of heat by calculating the entropy change first. This is the approach we follow in this example.

By first law

$$\Delta U_{AB} = W + Q.$$

Applying eq. (4.7) and noting that temperature remains constant through the process, we obtain

$$dQ = TdS \quad \Rightarrow \quad Q = \int_A^B TdS = T \int_A^B dS = T(S_B - S_A).$$

From steam tables we find

$$\begin{aligned} U_A &= 2940.9 \text{ kJ/kg} & S_A &= 7.0375 \text{ kJ/kg K} \\ U_B &= 2920.6 \text{ kJ/kg} & S_B &= 6.7712 \text{ kJ/kg K}. \end{aligned}$$

Therefore,

$$\begin{aligned} \Delta U_{AB} &= -20.3 \text{ kJ/kg} \\ Q &= (400 + 273.15)(6.7712 - 7.0375) = -179.26 \text{ kJ/kg} \\ W &= \Delta U_{AB} - Q = -20.3 + 179.26 = 158.96 \text{ kJ/kg}. \end{aligned}$$

There is another way to do this problem: since the process is reversible process, the  $PV$  work may be calculated as

$$W_{\text{rev}} = - \int_A^B PdV.$$

This integration may be done numerically, as was done in Example 3.3. Once the work is known, heat may be calculated from the energy balance. This procedure is more calculation-intensive however. The solution that utilizes entropy is much simpler in this case.