Expansion

Example 3.17: Reversible Adiabatic ~~Compression~~ of Nitrogen

Nitrogen expands reversibly in an insulated cylinder fitted with a piston from 500 K, 5 bar to a final pressure of 1 bar. Determine the final temperature as well as the amount of work assuming nitrogen to be in the ideal-gas state. The heat capacity of nitrogen is

$$C_P^{\text{ig}}/R = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4$$

with T in kelvin and

$$a_0 = 3.539,\ \ a_1 = -0.000261,\ \ a_2 = 7.0 \times 10^{-8},\ \ a_3 = 1.57 \times 10^{-9},\ \ a_4 = -9.9 \times 10^{-13}.$$

Solution The relationship between pressure and temperature in the initial and final states is given by eq. (3.44). Solving for T_2,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{R/\overline{C}_{P\log}^{\text{ig}}}. \tag{A}$$

Since the logarithmic mean capacity depends on T_2, this equation must be solved by trial and error. We start by assuming $\overline{C}_{P\log}^{\text{ig}}$ to be equal to the C_P^{ig} at the initial temperature, $T_1 = 500$ K:

$$\frac{C_P^{\text{ig}}(500\ \text{K})}{R} = 3.560.$$

Using this value in eq. (A), the temperature is

$$T_2 = (500\ \text{K}) \left(\frac{1\ \text{bar}}{5\ \text{bar}}\right)^{1/3.560} = 318.164\ \text{K}.$$

We now calculate the logarithmic mean capacity between 500 K and 318.164 K by integration of the equation for C_P^{ig}. For the polynomial expression given here, the result is

$$\frac{\overline{C}_{P\log}^{\text{ig}}}{R} = \frac{a_0 \ln \dfrac{T_2}{T_1} + a_1(T_2 - T_1) + \dfrac{a_2}{2}\left(T_2^2 - T_1^2\right) + \dfrac{a_3}{3}\left(T_2^3 - T_1^3\right) + \dfrac{a_4}{4}\left(T_2^4 - T_1^4\right)}{\ln(T_2/T_1)} = 3.524.$$

Using this value we calculate a revised temperature from eq. [A]:

$$T_2 = (500\ \text{K}) \left(\frac{1\ \text{bar}}{5\ \text{bar}}\right)^{1/3.524} = 316.697\ \text{K}.$$

This procedure is repeated until the temperature does not change any more. These iterations are summarized below:

Iteration	C_P^{ig}/R	T_B (K)
1	3.560	318.164
2	3.524	316.697
3	3.524	316.688
4	3.524	316.688

Therefore, $T_B = 316.688$ K.