where the latter estimates, over a time duration N, are based on a zero-mean ergodic assumption. Given our assumed quantizer range $2x_{\max}$ and a quantization interval $\Delta = \frac{2x_{\max}}{2^B}$ for a B-bit quantizer, and the uniform pdf, it is possible to show that (Exercise 12.2)

$$\begin{aligned}\sigma_e^2 &= \frac{\Delta^2}{12}\\ &= \frac{(2x_{\max}/2^B)^2}{12} = \frac{x_{\max}^2}{(3)2^{2B}}.\end{aligned} \tag{12.3}$$

We can then express the SNR as

$$\begin{aligned}\text{SNR} &= \frac{\sigma_x^2}{\sigma_e^2}\\ &= \sigma_x^2\left(\frac{(3)2^{2B}}{x_{\max}^2}\right) = \frac{(3)2^{2B}}{(x_{\max}/\sigma_x)^2}\end{aligned}$$

or in decibels (dB) as

$$\begin{aligned}\text{SNR(dB)} &= 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right)\\ &= 10\big(\log_{10}3 + 2B\log_{10}2\big) - 20\log_{10}\left(\frac{x_{\max}}{\sigma_x}\right)\\ &\approx 6B + 4.77 - 20\log_{10}\left(\frac{x_{\max}}{\sigma_x}\right).\end{aligned} \tag{12.4}$$

Because $x_{\max} = 4\sigma_x$, then

$$\text{SNR(dB)} \approx 6\text{B} - 7.2$$

and thus each bit contributes 6 dB to the SNR.

This simple uniform quantization scheme is called *pulse code modulation* (PCM) [33],[71]. Here B bits of information per sample are transmitted as a codeword. The advantages of the scheme are that it is *instantaneous*, i.e., there is no coding delay, and it is not signal-specific, e.g., it does not distinguish between speech and music. A disadvantage is that at least 11 bits are required for "toll quality," i.e., equivalent to typical telephone quality. For a sampling rate of 10000 samples/s, for example, the required bit rate is $I = (11 \text{ bits}) \times (10000 \text{ samples/s})$ = 110,000 bps in transmission systems.

EXAMPLE 12.2 Consider a compact disc (CD) player that uses 16-bit PCM. This gives a SNR $= 96 - 7.2$ dB $= 88.8$ dB for a bit rate of 320,000 bps. This high bit rate is not of concern because space is not a limitation in this medium. ▲

Although uniform quantization is quite straightforward and appears to be a natural approach, it may not be optimal, i.e., the SNR may not be as small as we could obtain for a certain number of decision and reconstruction levels. To understand this limitation, suppose that the amplitude of $x[n]$ is much more likely to be in one particular region than in another, e.g., low values occurring much more often than high values. This certainly is the case for a speech signal, given the speech pdf of Figure 12.2. Large values do not occur relatively often, corresponding to a very