and, therefore,

$$h[n] = D_*^{-1}(\hat{y}[n])$$

thus separating the impulse response. ▲

An approach for finding the components of the canonical representation and, in particular, the elements $D_*$ and $D_*^{-1}$, is to note that if $x[n] = x_1[n] * x_2[n]$, then the $z$-transform of $x[n]$ is given by $X(z) = X_1(z)X_2(z)$ and because we want the property that convolution maps to addition, i.e., $D_*(x_1[n] * x_2[n]) = D_*(x_1[n]) + D_*(x_2[n])$, this motivates the use of the logarithm and complex exponential in the operators; i.e., $D_*[x] = \log(Z[x])$ and $D_*^{-1}[x] = Z^{-1}[\exp(x)]$ where $Z$ denotes $z$-transform. However, if we want to represent sequences in the time domain, rather than in the $z$ domain, then it's desirable to have the operations $D_* = Z^{-1}[\log(Z)]$ and $D_*^{-1} = Z^{-1}[\exp(Z)]$. The canonical system with the forward and inverse operators is summarized in Figure 6.4, showing that our selection of $D_*$ and $D_*^{-1}$ gives the desired properties of mapping convolution to addition and addition back to convolution, respectively. However, in this construction of $D_*$ we have overlooked the definition of the logarithm of a complex $z$-transform which we refer to henceforth as the "complex logarithm." Because the complex logarithm is key to the canonical system, the existence of $D_*$ relies on the validity of $\log[X_1(z)X_2(z)] = \log[X_1(z)] + \log[X_2(z)]$ and this will depend on

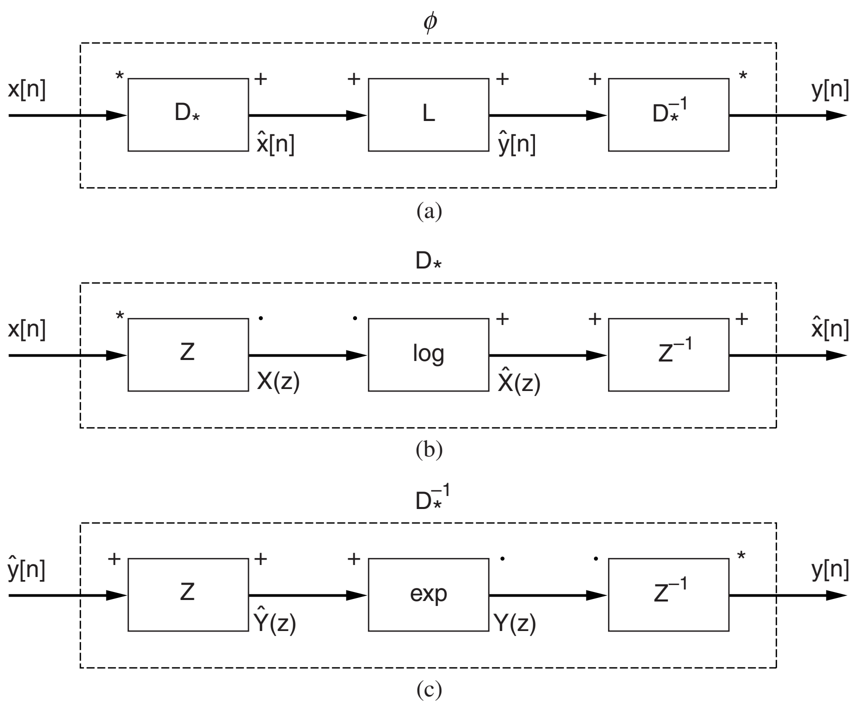


**Figure 6.4** Homomorphic system for convolution: (a) canonical formulation; (b) the subsystem $D_*$; and (c) its inverse.

SOURCE: A.V. Oppenheim and R.W. Schafer, *Discrete-Time Signal Processing* [13]. ©1989, Pearson Education, Inc. Used by permission.