a short-time rectangular window of about 40 ms to various speech waveforms. The autocorrelation function for the fricative phone is noisy and non-periodic as well as impulse-like, while for the voiced phone it is periodic-like with decreasing amplitude. The autocorrelation function of the voiced plosive shows a high-frequency and a noise contribution superimposed on a periodic component, while its unvoiced counterpart is comprised mainly of a high-frequency component with overriding noise due to the turbulent source and aspiration following the burst. ▲

With our new definitions, by letting $\Phi_n[i,k] = r_n[i-k]$, we can rewrite the normal equations as

$$\sum_{k=1}^{p} \alpha_k r_n[i-k] = r_n[i-0], \quad 1 \le i \le p$$

or

$$\sum_{k=1}^{p} \alpha_k r_n[i-k] = r_n[i], \quad 1 \le i \le p$$

which represent p linear equations in p unknowns, α_k, for $1 \le k \le p$. Using the normal equations solution, it can then be shown that the corresponding minimum mean-squared prediction error is given by

$$E_n = r_n[0] - \sum_{k=1}^{p} \alpha_k r_n[k]. \tag{5.15}$$

As before, the normal equations can be put in matrix form:

$$R_n \underline{\alpha} = \underline{r}_n, \tag{5.16}$$

that is,

$$\underbrace{\begin{bmatrix} r_n[0] & r_n[1] & r_n[2] & \dots & r_n[p-1] \\ r_n[1] & r_n[0] & r_n[1] & \dots & r_n[p-2] \\ r_n[2] & r_n[1] & r_n[0] & \dots & r_n[p-3] \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ r_n[p-1] & r_n[p-2] & r_n[p-3] & & r_n[0] \end{bmatrix}}_{R_n} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_p \end{bmatrix}}_{\underline{\alpha}} = \underbrace{\begin{bmatrix} r_n[1] \\ r_n[2] \\ \cdot \\ \cdot \\ \cdot \\ r_n[p] \end{bmatrix}}_{\underline{r}_n}.$$

The matrix R_n in Equation (5.16) has the property of being symmetric about the diagonal and all elements of the diagonal are equal. We will see that this structure, which is referred to as the "Toeplitz" property, implies an efficient algorithm for solution. Motivation for the name "autocorrelation method" now becomes clear; entries in the matrix R_n are the first p autocorrelation coefficients of $s_n[m]$. It can be shown that the columns of R_n are linearly independent and thus R_n is invertible [16],[25].