so that

$$E^i(z) = E^{i-1}(z) - k_i z^{-i} B^{i-1}(z) \tag{5.60}$$

which, in the time domain, is given by

$$e^i[m] = e^{i-1}[m] - k_i b^{i-1}[m-1]. \tag{5.61}$$

Now, substituting Equation (5.58) into Equation (5.55),

$$\begin{aligned} B^i(z) &= z^{-i}\left[A^{i-1}(z) - k_i z^i A^{i-1}(z)\right] S(z) \\ &= z^{-i} A^{i-1}(z^{-1}) S(z) - k_i A^{i-1}(z) S(z) \\ &= z^{-1} \underbrace{z^{-(i-1)} A^{i-1}(z^{-1}) S(z)}_{B^{i-1}(z)} - k_i \underbrace{A^{i-1}(z) S(z)}_{E^{i-1}(z)} \end{aligned}$$

or

$$B^i(z) = z^{-1} B^{i-1}(z) - k_i E^{i-1}(z) \tag{5.62}$$

which, in the time domain, is given by

$$b^i[m] = b^{i-1}[m-1] - k_i e^{i-1}[m]. \tag{5.63}$$

EXERCISES

5.1 Show that for the autocorrelation method of linear prediction of Section 5.3.3, the normal equations are written as

$$\sum_{k=1}^{p} \alpha_k \Phi_n(i,k) = \Phi_n(i,0), \qquad i = 1, 2, 3 \dots p$$

where

$$\Phi_n(i,k) = \sum_{m=0}^{N+p-1} s_n[m-i] s_n[m-k], \qquad 1 \le i \le p, \qquad 0 \le k \le p$$

and where N_w is the window length.

5.2 In this problem, you show that if $s_n[m]$ is a segment of a periodic sequence, then its autocorrelation $r_n[\tau]$ is periodic-like. You also investigate a small deviation from periodicity. In particular, consider the periodic impulse train

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - kP].$$

(a) Compute and sketch the autocorrelation $r_n[\tau]$ of the windowed sequence $x[n]$, i.e., $x[n]w[n]$, when the window applied to $x[n]$ is rectangular over the interval $0 \le n < N_w$ with length $N_w = 4P$, showing that $r_n[\tau]$ is periodic-like and falls off roughly linearly with increasing τ.

(b) How does your result from part (a) change if the pitch period increases by one sample on each period and the window length is long enough to cover the first three impulses of $x[n]$? Observe that the first three impulses occur at $n = 0$, $P+1$, and $2P+2$.