S3: Using the result of step **S2**, update coefficients for the ith pole model as

$$\alpha_i^i = k_i$$
$$\alpha_j^i = \alpha_j^{i-1} - k_i \alpha_{i-j}^{i-1}, \qquad 1 \leq j \leq i-1$$

where we have set the highest-order coefficient to k_i and where k_i has also been used as a weighting factor on the coefficients of order $i-1$ to obtain the coefficients of order i. We refer to the constants k_i as the *partial correlation coefficients* for which the motivation will shortly become clear. These coefficients are also called the *PARCOR* coefficients.

S4: Update the minimum mean-squared prediction error for the ith pole model as

$$E^i = (1 - k_i^2)E^{i-1}.$$

E^i, being a mean-squared prediction error, provides a way of monitoring the accuracy of prediction when the predictor order is unknown.

S5: Repeat steps **S2** to **S4** for $i = 1, 2, \ldots p$. At the pth step we have the desired pth order predictor coefficients

$$\alpha_j^* = \alpha_j^p, \quad 1 \leq j \leq p$$

where "*" denotes the (optimal) coefficients that minimize the mean-squared pth-order prediction error.

The derivation of the Levinson recursion can be found in [15],[16]. From step **S2**, we can show on each iteration that the predictor coefficients α_k can be written as solely functions of the autocorrelation coefficients (Exercise 5.11). Finally, the desired transfer function is given by

$$H(z) = \frac{A}{1 - \sum_{k=1}^{p} \alpha_k^* z^{-k}}$$

where the gain A has yet to be determined. Calculation of the gain depends on certain properties of the estimated prediction coefficients and will be described later in this section.

Properties — Properties of the autocorrelation method motivated by the Levinson recursion are given below.

P1: The magnitude of each of the partial correlation coefficients is *less than unity*, i.e., $|k_i| < 1$. To show this, we first recall that the mean-squared prediction error is always greater than zero because we cannot attain perfect prediction even when the sequence follows an all-pole model. Therefore, from the Levinson recursion, we have

$$E^i = (1 - k_i^2)E^{i-1} > 0$$

from which

$$(1 - k_i^2) = \frac{E^i}{E^{i-1}} > 0$$

and so it follows that

$$k_i^2 < 1 \quad \text{or} \quad |k_i| < 1.$$