Equation (2.14) from Chapter 2, i.e.,

$$B = \int_{-\infty}^{\infty} \left(\frac{da(t)}{dt} \right)^2 dt + \int_{-\infty}^{\infty} \left(\frac{d\phi(t)}{dt} - \bar{\omega} \right)^2 a^2(t)dt \tag{4.54}$$

(with $\bar{\omega}$ the mean frequency) Cohen [5] proposed a definition of *instantaneous bandwidth* that is dependent *only* on the amplitude of $x(t)$ and not its frequency, i.e.,

$$BW(t) = \frac{1}{2\pi} \left(\frac{a'(t)}{a(t)} \right).$$

Show in the special case of a decaying exponential, i.e.,

$$a(t) = e^{-\alpha t},$$

that Cohen's definition of instantaneous bandwidth becomes

$$BW(t) = \frac{\alpha}{2\pi}. \tag{4.55}$$

A more standard definition of bandwidth is the frequency distance between spectrum samples that are 3 dB lower than at the spectral peak. Show that the expression in Equation (4.55) is one-half of this standard definition. Likewise, again motivated in part by the bandwidth definition in Equation (4.54), as described in Chapter 2, Cohen defined $\frac{d\phi(t)}{dt}$ as the *instantaneous frequency* of $x(t)$. Describe qualitatively how the bandwidth and frequency multiplier factors in part (d) of this problem affect the bandwidth defined in Equation (4.54). In your description, consider how a rapid truncation corresponds to a larger instantaneous bandwidth $BW(t)$ and assume that $\frac{d\phi(t)}{dt}$ roughly follows the time-varying frequency $\Omega_1(t)$.

4.20 (MATLAB) The bilinear transformation used in mapping a Laplace transform to a z-transform is given by the s-plane to z-plane mapping of $s = \frac{2}{T}[\frac{z+1}{z-1}]$ and has the property that the imaginary axis in the s-plane maps to the unit circle in the z-plane [22]. The constant T is the sampling interval. Use the bilinear transformation to obtain the z-transform representation of the continuous-time radiation load of Equation (4.24). Specifically, with $T = 10^{-4}$, $c = 350$ m/s, $R_r = 1.4$, and $L_r = (31.5)10^{-6}$, argue that $Z_r(z) \approx 1 - z^{-1}$ where any fixed scaling is ignored. Use MATLAB to plot the magnitude of $Z_r(z)$ on the unit circle and show that $Z_r(z)$ introduces about a 6 dB/octave highpass effect. Argue from both a time- and frequency-domain viewpoint that the impulse response associated with $Z_r(z)$ acts as a differentiator.

4.21 (MATLAB)[20] In his work on nonlinear modeling of speech production, Teager [30] used the following "energy operator" on speech-related signals $x[n]$:

$$\psi(x[n]) = x^2[n] - x[n-1]x[n+1]. \tag{4.56}$$

Kaiser [11] analyzed ψ and showed that it yields the energy of simple oscillators that generated the signal $x[n]$; this energy measure is a function of the frequency as well as the amplitude composition of a signal. Kaiser also showed that ψ can track the instantaneous frequency in single sinusoids and chirp signals, possibly exponentially damped. Teager applied ψ to signals resulting from bandpass filtering of speech vowels in the vicinity of their formants. The output from the energy operator fre-

[20] This problem can be considered a preview to Section 11.5.