Applying the two boundary conditions with Equation (4.27), we then have

$$u_k^+\left(t - \frac{l_k}{c}\right) - u_k^-\left(t + \frac{l_k}{c}\right) = u_{k+1}^+(t) - u_{k+1}^-(t)$$

$$\frac{A_{k+1}}{A_k}\left[u_k^+\left(t - \frac{l_k}{c}\right) + u_k^-\left(t + \frac{l_k}{c}\right)\right] = u_{k+1}^+(t) + u_{k+1}^-(t). \tag{4.28}$$

Now define $\tau_k = \frac{l_k}{c}$, which is the time of propagation down the length of the tube. Then the first equation in the Equation (4.28) pair can be written as

$$u_k^-(t + \tau_k) = u_k^+(t - \tau_k) - u_{k+1}^+(t) + u_{k+1}^-(t)$$

and substituting this expression for $u_k^-(t + \tau_k)$ into the second equation of our pair, we obtain, after some algebra,

$$u_{k+1}^+(t) = \left[\frac{2A_{k+1}}{A_{k+1} + A_k}\right]u_k^+(t - \tau_k) + \left[\frac{A_{k+1} - A_k}{A_{k+1} + A_k}\right]u_{k+1}^-(t). \tag{4.29}$$

Then subtracting the top from the bottom component of the above modified equation pair, we obtain, after some rearranging,

$$u_k^-(t + \tau_k) = -\left[\frac{A_{k+1} - A_k}{A_{k+1} + A_k}\right]u_k^+(t - \tau_k) + \left[\frac{2A_k}{A_{k+1} + A_k}\right]u_{k+1}^-(t). \tag{4.30}$$

Equations (4.29) and (4.30) illustrate the general rule that at a discontinuity along x in the area function $A(x, t)$ there occur *propagation and reflection of the traveling wave*, and so in each uniform tube, part of a traveling wave propagates to the next tube and part is reflected back (Figure 4.15). This is analogous to propagation and reflection due to a change in the impedance along an electrical transmission line.

From Equation (4.29), we can therefore interpret $u_{k+1}^+(t)$, which is the forward-traveling wave in the $(k + 1)$st tube at $x = 0$, as having two components:

1. A portion of the forward-traveling wave from the previous tube, $u_k^+(t - \tau_k)$, propagates across the boundary.
2. A portion of the backward-traveling wave within the $(k+1)$st tube, $u_{k+1}^-(t)$, gets reflected back.

Observe that $u_k^+(t - \tau_k)$ occurs at the $(k + 1)$st junction and equals the forward wave $u_k^+(t)$, which appeared at the kth junction τ_k seconds earlier. This interpretation is applied generally to the forward/backward waves at each tube junction. Likewise, from Equation (4.30), we can interpret $u_k^-(t + \tau_k)$, which is the backward-traveling wave in the kth tube at $x = l_k$, as having two components:

1. A portion of the forward-traveling wave in the kth tube, $u_k^+(t - \tau_k)$, gets reflected back.
2. A portion of the backward-traveling wave within the $(k + 1)$st tube, $u_{k+1}^-(t)$, propagates across the boundary.

Figure 4.15 further illustrates the forward- and backward-traveling wave components at the tube boundaries.