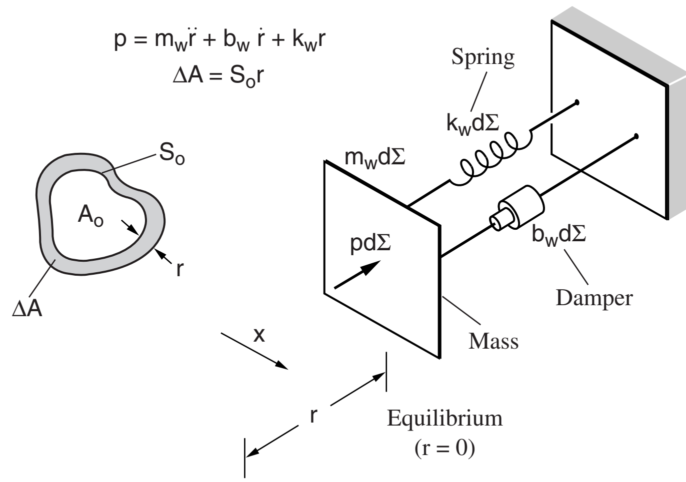

Figure 4.9 Mechanical model of differential surface element $d\sum$ of vibrating wall. Adjacent wall elements are assumed uncoupled.

SOURCE: M.R. Portnoff, *A Quasi-One-Dimensional Digital Simulation for the Time-Varying Vocal Tract* [26]. ©1973, M.R. Portnoff and the Massachusetts Institute of Technology. Used by permission.

Portnoff has solved these coupled equations using standard numerical simulation techniques[12] invoking finite-difference approximations to the spatial partial differential operator $\frac{\partial}{\partial x}$. Suppose that the length of the tube is l. Then we seek the acoustic pressure and volume velocity at N equally spaced points over the interval $[0, l]$. In particular, $\frac{\partial}{\partial x}$ was approximated by a central difference with averaging; that is, for a partial differential with respect to x of a function $f(x)$, first compute the central difference over a spatial difference $\Delta x = \frac{l}{N-1}$:

$$g[n] = \frac{1}{2\Delta x}(f[(n-1)\Delta x] - f[(n+1)\Delta x])$$

and then perform a first backward average:

$$\frac{\partial}{\partial x} f(x) \approx \frac{1}{2}(g[n\Delta x] + g[(n-1)\Delta x]).$$

It can be shown that this transformation from the continuous- to the discrete-space variable is equivalent to a bilinear transformation.[13] This partial differential approximation leads to

[12] We can also obtain in this case an approximate closed-form, frequency-domain solution [26],[28], but we choose here to describe the numerical simulation because this solution approach is the basis for later-encountered nonlinear coupled equations.

[13] The bilinear transformation for going from continuous to discrete time is given by the s-plane to z-plane mapping of $s = \frac{2}{\Delta t}[\frac{z+1}{z-1}]$ and has the property that the imaginary axis in the s-plane maps to the unit circle in the z-plane [22]. The frequency Ω in continuous time is related to the discrete-time frequency by the tangential mapping $\frac{\Delta t \Omega}{2} = \tan(\frac{\omega}{2})$. Likewise, the bilinear mapping in going from continuous space to discrete space is given by $K = \frac{2}{\Delta x}[\frac{k+1}{k-1}]$ and has the same tangential mapping as the time variable.