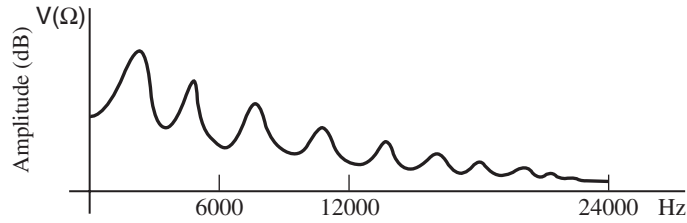


time sampling interval is  $T = 2\tau' = l/c$ . Sketch the discrete-time frequency response from the source to the lips. Deduce the discrete-time transfer function. How many resonances are now represented over the frequency interval  $[0, \pi]$ ?

- (d) Find a continuous-time impulse response with bandwidth  $\pi/T$ , that, when sampled with sampling interval  $T$  of part (c), gives the discrete-time frequency response [of part (c)] over the discrete-frequency interval  $[-\pi, \pi]$ . How does this continuous-time impulse response differ from the true continuous-time impulse response of the tube? How does it differ from the true impulse response at the time samples  $nT = n2\tau'$ ? How many equal-length tubes (same cross-section) of a concatenated tube model do you need as a basis for the resulting discrete-time model to represent the true impulse response for all time?
- (e) Suppose now that energy loss takes place in a spatially-varying tube, and its frequency response, given in Figure 4.36, is bandlimited to 24000 Hz. Suppose all energy loss occurs at the output radiation load, and suppose that the spatially-varying tract is approximated by a concatenated-tube model. How many equal-length tubes are required in a discrete-time model to represent the frequency response over only the first 12000 Hz?



**Figure 4.36** Lossy tube frequency response.

**4.19** In this problem you will fill in some of the missing steps in Section 4.5 on vocal fold/vocal tract interaction and the “truncation” effect. We saw in Figure 4.22 an electrical analog of a simple model for airflow through the glottis.  $P_{sg}$  at the left is the subglottal pressure in the lungs that is the power source for speech,  $p(t)$  is the sound pressure corresponding to a single first formant, and  $Z_g(t)$  is the time-varying impedance of the glottis, defined as the ratio of *transglottal pressure* across the glottis,  $p_{tg}(t)$ , to the *glottal volume velocity* through the glottis,  $u_g(t)$ .

- (a) Writing the current nodal equation of the circuit in Figure 4.22, and using Equation (4.43), show that

$$C \frac{dp(t)}{dt} + \frac{p(t)}{R} + \frac{1}{L} \int_0^t p(\tau) d\tau = A(t) \sqrt{\frac{2p_{tg}(t)}{k\rho}}. \quad (4.50)$$

- (b) Differentiate Equation (4.45) with respect to time and show that

$$C \frac{d^2 p(t)}{dt^2} + \left[ \frac{1}{R} + \frac{1}{2} \dot{g}_o(t) \right] \frac{dp(t)}{dt} + \left[ \frac{1}{L} + \frac{1}{2} \dot{g}_o(t) \right] p(t) = \dot{u}_{sc}(t). \quad (4.51)$$

Then argue from Equation (4.51) that Figure 4.24 gives the Norton equivalent circuit to the original circuit of Figure 4.22, where the equivalent time-varying resistance and inductance