CHAPTER

1

# A Philosophical Viewpoint

## 1.1 THE "ULTIMATE" MEASUREMENT

Let's begin with a *warmup* exercise for the imagination. Forget the reality of the world around you for a few moments and imagine you have been transported into some *new* world, with which you are completely unfamiliar. We will assume you are well versed in mathematics, physics, and the scientific method and are accustomed to making observations, postulating theories to fit the observations, and then testing these theories with new and different observations. You have been assigned the task of *characterizing* this new world in some way, so you can subsequently explain how it works to other people. Let's also assume you have plentiful resources at your disposal, so you can make all manner of observations and utilize unlimited computing power whenever needed. How might you try to accomplish this demanding task?

As a first step, you collect a vast amount of quantitative *data,* based on your *observations* of things that seem to be happening in this new world. Initially, things may seem to make little sense, but gradually patterns begin to emerge. Some conditions always appear to lead to the same results, and so become rather predictable. You might even go so far as to postulate a few tentative "laws of nature" that seem to prevail. You can then use these newly established "laws" to predict results you expect under some *new* set of conditions. Your tentative theories may fail at times, but often they will make reasonably correct predictions, and you will feel encouraged to continue observing and recording subsequent events that occur. There will always be *some* rogue observations that don't quite fit your theories, but of course that is not surprising, because you knew ahead of time your theories would not be perfect.

Let's digress for a moment. Geometries contain *invariant* quantities called *tensors,* which are independent of the coordinate system used in their representation. There are both *covariant* and *contravariant* tensors, distinguished by their equations of transformation between different coordinate systems. If you choose a *new* coordinate system, then each tensor will have *new* component *values,* but these new values can be determined from the *original* values, in conjunction with the transformation equations that *define* the new coordinate system (in terms of the old one), with absolutely *no* dependence on the attributes of the tensor itself. For example, you can express the location of a fixed point in a plane using either rectangular or polar coordinates. The corresponding coordinate *values* are, of necessity, *different,* reflecting the fact that the point is actually *fixed* in its physical location, but there exist two different, yet precisely related, ways to describe this location.

These tensors or invariants form the *foundation* of any geometry. For example, there is usually a metric tensor that describes the "distance" between any two points in the system. There may be a curvature tensor, a torsion tensor, and assorted other tensors that further describe the characteristics of the space. In the model of our *home* world, there are energy-momentum tensors, electromagnetic field tensors, and stress and strain tensors, among others, in addition to a metric tensor and a curvature tensor (which describes the effects of gravitational fields and acceleration).

There is also a fundamental question of the *dimensionality* of the space you are attempting to characterize. Is the space two-dimensional, three-dimensional, four-dimensional, or of *higher* dimensionality? How can you tell? People in our home world thought it was a three-dimensional place at one time, but after further observations they decided it was probably at least four-dimensional. They observed that the speed of light was an *invariant* with respect to transformations between coordinate systems that were moving at a *uniform velocity* relative to one another. The speed of light is a *scalar* quantity and is a tensor of order *zero.* However, a four-dimensional space is needed to account for this seemingly strange behavior, with *imaginary time* as the fourth coordinate.

After further study, people concluded this four-space was *curved* instead of flat, because that assumption seemed to best describe the behavior of events represented in *relatively accelerating* coordinate systems, and in the presence of *gravitational fields* associated with masses. These observations of our home world were made by a number of people, but it was largely Albert Einstein who constructed the theories of (special and general) relativity to explain these observations (circa 1905–1916). At the present time, considerable effort is being devoted to constructing some sort of *unified field theory,* which would explain all of the force fields we know about (gravitational, electromagnetic, and the weak and strong nuclear forces), along with all of the effects predicted by *quantum field theory,* in a single fundamental theory of "everything." Some of these theories require rather large dimensionalities (like ten, eleven, or twenty-six, for example!). We have *yet* to decide on the dimensionality of the space in which we live, or even to construct a comprehensive *model* of this space, even though we have countless observations on which to build.

Returning our attention to the task at hand, one of the techniques you *should* employ to characterize this new world is to establish a mental (or abstract) *model* that seems to adequately describe all of your physical observations, but to keep this model *separate* in your mind from what you perceive as physical *reality.* In other words, always maintain two *parallel* pictures or representations of the world. One picture comprises all of the *observations* that have been made of the physical world, and the other picture comprises a *mathematical (or geometrical) model,* designed to represent the way the world *seems* to work.

Since there are *invariants* in the physical world, such as the conservation of energy/momentum, the velocity of light, and the equivalence between gravitation and acceleration, it seems logical to use a mathematical model having a set of invariant tensors, and then to try to establish a *one-to-one* correspondence between the invariants in these two domains. Keep in mind that the only *connections* between these two domains are the observations of real physical events, in comparison to observations of corresponding events from the model. These *observations* from the two domains comprise the "glue" that couples the physical world with our *model* of the world.

The measurement task is to adjust the *parameters* in your mental model so it (at least approximately) predicts the *same* results you actually observe in the physical world, under "identical" conditions. Using the model, you can then make *new* predictions of things you expect to observe in the real world, under some *new* set of conditions, or with *new* input excitations. If you actually observe these results, then the validity of your model is strengthened. If your observations contradict the predictions of the model, then *modify* the model and try again (assuming your observations are indeed *correct*).

In this process, you never allow yourself to *equate* the mathematical model with physical reality, or vice versa. There is no reason to believe the physical world even *entails* any mathematical relationships, since *mathematics* is strictly a human invention. The only link between these two pictures of the world is via the observed versus the predicted outcome, for each test case you consider. The various rules, relationships, and mathematical quantities that exist in your model may or may not correspond to *anything* that exists in the physical world. Always remember your model of the world is an approximation and an abstraction. It is always in a state of *flux* because you continually upgrade the model as new observations accumulate.

This hypothetical task of measuring the parameters that describe some new world, is the "*mother* of all measurements," in comparison to which, *other* measurements seem relatively simple! In fact, *any* other measurement job, by definition, must comprise a *subset* of this world measurement task. However, the principles and procedures are the *same,* no matter what size the measurement assignment might appear to be. In each case, collect as much raw physical data as practical, and by some means, construct a good mental (meaning mathematical) model of the thing you are attempting to measure. With your mathematical skills, design an algorithm that will generate some sort of "best" estimates of the corresponding model *parameters,* such that the predicted output from your model closely *matches* the observed output from the physical system you are observing. Keep in mind that the definition of "*best*" is rather tenuous and will depend on your judgment to some extent. Measuring is something of an *art,* even though there are many aspects that can be described with mathematical precision.

## 1.2 THE MINDSET OF MEASURING

When you look at an ordinary rectangular table, you see an object with a width, a length, and an area. You could say, "The table is 3 feet wide and 6 feet long, with an area of 18 square feet." However, a more *apt* depiction is, "The table is being *modeled* as a *rectangle* having two *parameters,* width and length (from which the area can be calculated). The *best* parameter estimates, to date, are 3 feet for the width and 6 feet for the length." This latter description separates the physical object itself from the parameters in your mental picture of the object. In fact, nobody really knows the exact dimensions of the table, or that it is a *plane* object, or even a *rectangular* object. It is virtually impossible for the table to be *exactly* flat, and to have *exactly* parallel sides and square corners! On a molecular scale, the boundary of the table will comprise a rough irregular contour rather than a set of four straight lines. You simply *choose* a plane rectangular model because it seems like a good *working approximation* to the actual table shape. If that model proves to be deficient, then you stand ready to modify it, as needed.

It is natural to talk about the width, length, and area as *integral* parts of the physical table, because the table seems to actually *have* a width, length, and area, and they *seem* to be invariant with time. But the problem is, you have *no* way of knowing what the values of these quantities really *are,* nor that they actually *are* invariant with time. Your only option is to make repeated *observations* of the table dimensions, and then to estimate the most likely values of these rectangular parameters, perhaps by *averaging* the observations together. The table is always vibrating (because mechanical structures continually vibrate on *some* scale) and the dimensions of the table are always changing with temperature, atmospheric pressure, relative humidity, and so on. In addition, the resolution of your tape measure is limited, so you must use your best judgment for *each* reading of the tape. As a consequence of all of these variables, the observed table dimensions will be *different* every time you sample them.

Following our rule, we will maintain a clear mental separation between the *actual* dimensions of the table at any instant of time (which we intuitively feel must exist, but that we can never fully determine) and our best *estimates* (or *measurements*) of these dimensions, based on many repeated trials, using the model we have chosen. We never confuse the actual physical attributes of something with our mental model of those attributes. It might sometimes require a conscious effort to accept this idea, because most of us were never taught to make this distinction explicitly. One useful technique is to temporarily inhibit your senses (by closing your eyes, for example), so you are *only* aware of the model and are not distracted by any physical object. This separation rule permits you to simplify the measurement task by allowing *incomplete* or *approximate* models and to accommodate the *variations* you observe every time you repeat the measurement, without blaming your measuring instruments for faulty readings.

The concept is easier to comprehend when *hidden* or *implicit* model parameters are involved. For example, in the GPS (Global Positioning System) satellite navigation scheme, the desired user measurements are latitude, longitude, and altitude (referenced to the center of the Earth). You can calculate these quantities from the observed *differences* in arrival times between signals transmitted from at least four orbiting satellites, all having known positions relative to the Earth. In this example, the position coordinates of an observer are not necessarily associated with any local physical *object,* so it is very natural to think of them as *parameters* in a model of the Earth-satellite system.

## 1.3 MEASUREMENT QUALITY

If you are the end user of a measurement, you naturally expect results that are sufficiently accurate to use in *your* particular application. What does "sufficiently accurate" really mean? You are the user, so *you* must decide. In some cases, 10% errors might be tolerable, and in other cases you may require an error of only one part in $10^{12}$. If you decide to purchase a plot of land for your new house, you would not be very happy if the owner decided to establish the property boundaries by *stepping* off the distances involved, and you would also be reluctant to pay a surveyor to measure these distances to the nearest *quarter wavelength* of the light from a helium-neon laser! You want these distances to the nearest centimeter and *neither* to the nearest meter *nor* to the nearest micron. The "best" measurement for your application may not always be the one with the most accuracy and precision. However, regardless of your accuracy requirements, you *do* want some indication that the measurement actually *meets* your specifications. In many respects, the actual measured *value* is somewhat secondary to an analysis of all of the possible *errors* that might perturb that value. A comprehensive *error analysis* is the best way to be reasonably sure the measurement quality is adequate for your needs.

If you repeat a measurement in exactly the same way a million times, you will get a million different answers. A histogram of these values, divided by the total number of measurements, gives an approximation to a *probability density function.* A measurement is not fully characterized until its probability density function is also determined, either by direct measurement or by calculation from a model. A *single* measured value can fall at any point within the abscissa range of the probability density function, but you have *no* way of knowing where it *actually* falls. The best you can do is to measure or calculate a probability (some number between zero and one) that the measurement falls between *limits* or boundaries of your choosing.

In addition to random variations, there are often many assumptions that have been made about conditions that might affect the measured result. For example, any measure of distance is affected by temperature and atmospheric pressure and is further affected by any errors that may be built into the reference scale you have chosen. Other errors are introduced by the method you use to *compare* the unknown distance with the reference scale. Sometimes the very *act* of measuring a quantity will alter its value. For example, the measurement of an electrical voltage will depend on the amount of current drawn by the measuring instrument through the output impedance of the network under test, as well as by the intrinsic accuracy of the instrument itself. These *non*-random errors are called *biases* in the parameter estimates. Bias errors, unlike random errors, cannot be reduced by averaging additional data.

A measurement is no better than the reference scale you use. In general, the reference scale will be in error by some *multiplicative scale factor,* and it may also be *offset* from the true zero point. A voltmeter may read 10 millivolts when the actual input voltage is zero, and it may read something like 1.01 plus 10 millivolts when the actual input is 1 volt. Its internal voltage reference may be a standard cell, a Zener diode, or simply a metal spring working against the force of the magnetic field generated by the flow of current through a calibrated resistor. These reference scales are usually functions of temperature, atmospheric pressure, humidity, and even elapsed time. The zero point offset is usually sensitive to similar effects, but might further depend on the balance between two small offset voltages, such as is often found in operational amplifiers.

In addition, the reference scale subdivisions are never exactly *uniform,* so these errors will be transferred directly to the quantity being measured. If the voltage scale is only linear to 1% of full scale, then any subsequent voltage measurements are limited to that precision. There are also random errors due to the limits imposed by various sources of electrical noise involved in the comparison between the unknown and the internal reference voltage. Even if the unknown voltage is exactly *constant* (which it cannot be), this internal comparator noise causes each measurement to be different.

You may have noticed that few measuring instruments (including distance scales) seem to come from the manufacturer with their errors fully characterized, or even with any *mention* of their accuracy at all. Perhaps they expect you—the user—to believe their instruments are perfect and thus always make accurate measurements under all conditions! Perhaps they think the average user would be unnecessarily confused by extra error information! More likely, it is simply deemed to be too expensive and time consuming for the manufacturer to characterize all of the possible errors that could be introduced by the equipment, especially if the end users are not *asking* for this information. Perhaps the responsibility for this state of affairs ultimately rests with the *uncritical* end user. We are all familiar with the slogan, "Let the *buyer* beware," but we might also add, "Let the *measurer* beware!" If you do not *ask* for accuracy information, then the manufacturer may not bother to provide it for you.

## 1.4 ATTRIBUTES OF THE MEASUREMENT MODEL

One of the most useful applications of a model of a physical system is in *simulating* the behavior of that system, to avoid the need to physically *construct* the system from component parts. Perhaps even more useful is the ability to *vary* some of the model parameters, to determine the effects of these variations on the system performance. In an actual physical system, you seldom have access to *all* of the parameters, and you are seldom able to vary all of them to fully characterize their effects.

These attributes of a general physical model also apply to the model of a measurement. Some parameters have more effect on the output observations than others. It is very possible some parameters have virtually *no* effect on the observed data. With a good model of the system and the measurement configuration, it is possible to calculate these characteristics. This is called a *sensitivity analysis* of a system. If some of the parameters change, then you can calculate the effects on the output of the model. You can also calculate the contributions of each error mechanism or noise source, allowing an *error* *schedule* to be constructed, showing which error mechanisms are most important and which ones can perhaps be effectively ignored. By inverting this procedure, you can obtain the sensitivity of each *parameter value* relative to each noise source in the model. Furthermore, a good model will allow you to "adjust" the measured values, if current conditions differ significantly from the conditions under which previous measurements were made.

This modeling procedure has at least *one* rather unexpected and generally unwanted aspect that needs to be disclosed and discussed. Once a model has been selected, any measurements that result from its use tend to enforce a "self-fulfilling prophecy," in the sense that measurements will be *forced* to fit the chosen model, often without giving any indication that the model might not be *correct.* For example, if you postulate that an AC voltage source is to be modeled as a *constant* DC voltage, then your measurement will indeed indicate a constant average value (of zero volts). You may not notice the voltage actually fluctuating around this average value in a sinusoidal manner, with an amplitude of 120 volts AC rms (root-mean-square). You get an accurate measure of the parameter you *asked* for, even though your model is completely wrong! In this particular case, an additional measurement of the *variance* of the voltage samples would show something was wrong, but that may not always be sufficient. For another example, suppose you assume there are five dominant modes of vibration in a mechanical structure (over some frequency range), when in fact there are *six.* Your estimation algorithm may not be "smart" enough to notice this problem, and the resulting measurements will be of little use to anyone, because you simply cannot represent the behavior of six *physical* modes with only five *component* modeling modes at your disposal. A good measurement design includes variance estimates and should also include techniques to show that the number of parameters and the topology of the model are essentially correct.

A similar thing can happen when your chosen estimation algorithm has a built-in *bias* mechanism. You will often have *no clue* that this error exists, even though the magnitude of this bias error can be large enough to *swamp* the parameter being measured. This is a common problem with certain control system measuring methods in which some of the system noise is *squared* and is therefore *not* eliminated by subsequent averaging. Again, the measurement may be completely *invalid,* but you may accept it as completely *correct* because you have no evidence to the contrary. You may inadvertently base some very important decisions on this bogus result. A thorough study of the measurement model, complete with all noise sources *and* the parameter estimation algorithm, is needed to spot potential bias problems of this sort.

The measurement model *must* include all of the significant sources of noise or interference that could have an effect on the result. There are generally several noise sources within the system under test, and there are always random contributions from the measuring instrument itself. The importance of each of these sources of contamination depends on the parameters being measured, as well as on the measurement or estimation algorithm used to reduce the data. Some parameters may be very difficult or impossible to estimate under relatively high noise conditions. Sometimes a different estimation algorithm will yield better results. Any given reduction algorithm will generally work better for some parameters than for others in the model.

One of the most difficult tasks is to choose the proper *topology* for the model. If key elements are missing or misconnected, the measured parameter values will generally be different each time the system is excited in a different way, in which case you really don't have a *valid* measurement at *all.* If *extra* parameters are included, the estimation algorithm may have trouble assigning values to *any* of the parameters, even the ones that are correctly *placed* in the model. Ironically, the measurement algorithm will usually have *less* trouble with these topological errors if there is a small amount of noise in the system. It is difficult or impossible to make good measurements on a system having the *wrong* topology when the raw data is extremely "clean." However, with sufficient noise, these topological errors can sometimes be accommodated surprisingly well.

Having outlined the measurement process and the *mindset* required to put the measurement into proper perspective, we will proceed to discuss each aspect in more detail in the chapters that follow and will show several examples to illustrate these concepts in a practical way. Keep in mind that a measurement has *some* aspects that are very precisely defined by mathematical relationships, but there are *other* aspects that are very much an art and require the skill and judgment of everybody involved in the measurement task. Don't let the *apparent precision* of the various mathematical procedures intimidate you or give you a false sense of measurement quality. Good judgment and common sense are indispensable commodities in *all* measurement activities.

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