CHAPTER 3

Detection Regimes and Figures of Merit

Before proceeding to the next description of photodetectors, it is now appropriate to digress on system considerations concerning photodetection, to determine how good performance is in view of fundamental limits of sensitivity and speed of response, irrespective of the actual type of detector used. Other noise considerations will be developed in Chapter 6. Here, we anticipate a discussion of thermal and quantum regimes to point out the need for an internal gain in photodetectors. Also, we introduce the Jones figures of merit, which are frequently used in thermal IR detectors.

3.1 The Bandwidth-Noise Tradeoff

The problem of how to terminate the photodetector on a suitable load resistor, and trade-off performances between bandwidth and noise, is common to any quantum detector yielding emitted electrons (or internal electron-hole pairs) as a response to incoming photons. This is so not only in phototubes but also in photodiodes, CCD's, photoresistances, vidicon targets, etc., all of which are described by a current generator σP with a stray capacitance C across it.

Let us then consider the equivalent circuit of a quantum photodetector ending on a load resistor R, as shown in Fig.3-1, and evaluate its bandwidth and noise.

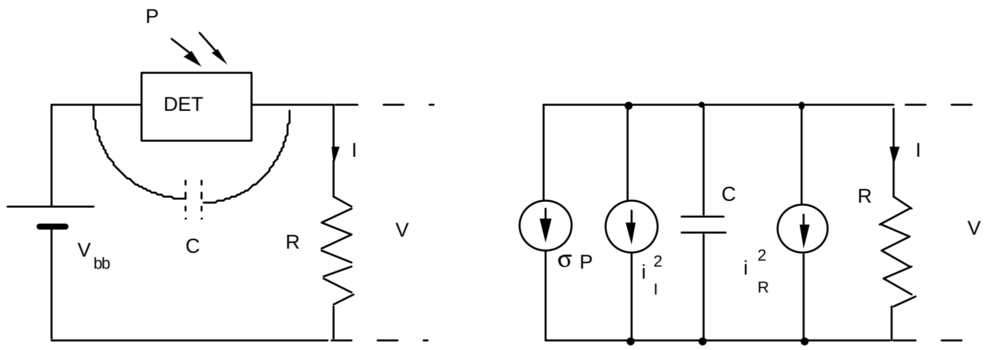

Figure 3-1 General circuit of a quantum photodetector (left) and its equivalent circuit including noise generators

The output signal, either in voltage $V=\sigma P{\cdot}R$ or in current $I=\sigma P$, has a 3-dB high-frequency cutoff given by:

$$B = 1/2\pi RC \tag{3.1}$$

Added to the signal, we find two noise contributions. One is the Johnson (or thermal) noise (see also Appendix A4) of the resistance R, with a quadratic mean value:

$$i^2_R = 4kTB/R \tag{3.2}$$

where k is the Boltzmann constant and T the absolute temperature.

The other noise is the quantum (or shot) noise associated with the discrete nature of the total current $I=I_{ph}+I_d$, the sum of the signal current and of the dark current. Its quadratic mean value is given by:

$$i^2_I = 2e\,(I_{ph}+I_d)\,B \tag{3.3}$$

where e is the electron charge and B, as in Eq.(3.2), is the observation bandwidth.

The two fluctuations are added to the useful signal, and so the corresponding noise generators are placed across the device terminals as in Fig.3-1. Since the two noises are statistically independent, their quadratic mean value combine to give the total fluctuation i^2_n as:

$$i^2_n = 2e\,(I_{ph}+I_d)\,B + 4kTB/R \qquad (3.4)$$

From Eqs.(3.1) and (3.4) we can see that bandwidth and noise impose opposite requirements on R: to maximize B one should use the smallest possible R, while to minimize i^2_n the largest possible R is required. Of course, the same conclusion would apply if, instead of the current I, we had chosen the voltage V across R.

A quantum photodetector, regarded in its basic form of photon-to-electron converter, cannot avoid this problem: it can have a good sensitivity using very high load resistances (up to GΩ's) as is actually done in some instrumentation applications requiring modest bandwidths (≈kHz or less), or it can be made fast by using low load resistances (e.g. R=50Ω), but at the expense of sensitivity.

To circumvent this trade-off, there are two approaches currently used, which we will describe in detail in the following chapters of this book:

(i) circuit-level solutions, providing an active two-terminal load or circuit exhibiting less noise than the Johnson limit applicable to any passive device. These may take the form of: a special termination, such as: *cold-resistance* and *switched capacitor*, or of a system recovery of bandwidth, as the *equalization* technique;

(ii) device-level solutions, providing a mechanism of internal gain that, by amplification of the photodetected current by a factor G (and of the quadratic noise fluctuations by G^2) before it is added to the Johnson noise generator, allows the use of a load resistance smaller by a factor G^2 at equal sensitivity.

3.2 QUANTUM AND THERMAL REGIMES

Let us now evaluate the relative weight of the two terms in Eq.(3.3). In general, we will have a good sensitivity performance when the shot noise is dominant compared to Johnson noise, i.e. for:

$$2e\,(I_{ph}+I_d)\,B \quad 4kTB/R$$

whence the condition:

$$R_{min} \quad (2kT/e)/(I_{ph}+I_d)$$
$$\quad 50\ \mathrm{mV}/(I_{ph}+I_d)\ \ [\text{at } T=300\ \mathrm{K}] \qquad (3.5)$$

At low signal levels, that is, for $I_{ph} << I_d$, very high values of resistance are required from Eq.(3.5); for example, R_{min}=10 GΩ for I_d=5 pA, not an unusually low dark current.

When we are using a value $R<R_{min}$ as the termination to the photodetector, the total noise can be written as:

$$i^2_n = 2e(I_{ph}+I_d)B+4kTB/R = [2e(I_{ph}+I_d)B](1+R_{min}/R) \tag{3.6}$$

or, the noise performance is degraded by a factor R_{min}/R compared to that intrinsic allowed by the dark current level.

In still other terms, using $R<R_{min}$ means that the shot-noise performance will be reached at a level of photocurrent not less than:

$$I_{ph}+I_d \quad (2kT/e)/R = 50 \text{ mV} / R \tag{3.7}$$

From Eq.(3.6) we see that for a fast photodiode with a 50Ω load this current is $I_{ph}+I_d$=1 mA, a very large value.

Let us now extend the above considerations to the calculation of the signal-to-noise ratio or S/N of a quantum photodetector with an (eventual) internal gain G. The mean output current is then:

$$I = \sigma P\, G = I_{ph}\, G \tag{3.8}$$

and the shot noise can be expressed, after Eq.(3.3), in the form:

$$i^2_I = 2e\,(I_{ph}+I_d)\, B\, G^2\, F \tag{3.3'}$$

where G^2 is the quadratic amplification of the noise, and F is the *excess noise factor* summarizing the eventual extra noise introduced by the amplification mechanism [in a non-amplified detector it is F=1 and G=1]. By adding to Eq.(3.3') the thermal noise given by Eq.(3.2), we can write the total noise as:

$$N = [2e\,(I_{ph}+I_d)\, B\, G^2\, F + 4kTB/R]^{1/2} \tag{3.9}$$

and from here, with S= I_{ph} G, we get for the S/N ratio:

$$S/N = \frac{I_{ph}}{[\,2e(I_{ph}+I_b)\, B\, F + 4kTB/RG^2]^{1/2}} \tag{3.10}$$

We can now introduce a critical value I_{ph0}, we call it*threshold of quantum regime*:

$$I_{ph0} = I_b + (2kT/e) / R\, F\, G^2 \tag{3.11}$$

Then, Eq.(3.10) becomes:

$$S/N = I_{ph} / [\,2e(I_{ph}+I_b)\, B\, F + 4kTB/RG^2]^{1/2} \tag{3.10'}$$

Two detection regimes are found, according to whether the signal I_{ph} is larger or smaller than I_{ph0}. For the signals, $I_{ph} > I_{ph0}$, and F=1, Eq.(3.10') becomes:

$$S/N = [I_{ph} / 2eB]^{1/2} \tag{3.12}$$

This S/N value is called the *quantum noise limit* of detection. It is important because it cannot be overcome by any detection system operating on coherent or incoherent radiation which obeys the Poisson photon statistics, of which Eq.(3.3) is a direct consequence.

The signal input power P_{ph0} at which the detector reaches the quantum-limited regime, using the spectral sensitivity σ, is:

$$P_{ph0} = I_{ph0} / \sigma = [I_b + (2kT/e)/RFG^2] / \sigma \tag{3.11a}$$

Let us remark that, for a digital transmission with bit period T, taking as the bandwidth the Nyquist value B=1/2T, and letting $N = (I_{ph} / e)T$, the (mean) number of photons detected per bit, Eq.(3.12) can be put in the very expressive form:

$$S/N = N^{1/2} \tag{3.13}$$

It is noteworthy that the quantum regime is not associated with the photodetector *itself*, as it is always reached at a sufficiently high signal level, $I_{ph} > I_{ph0}$; rather, a good sensitivity performance for a photodetector is indicated by a low value I_{ph0} (or P_{ph0}) at which the quantum limit is reached.

Now consider the small signal regime $I_{ph} < I_{ph0}$, for which the S/N ratio given by Eq.(3.10) becomes:

$$S/N = I_{ph} / [2eI_{ph0} B]^{1/2} \tag{3.14}$$

that is, the S/N ratio is proportional to the signal and the noise is constant and primarily given by the load resistance. This is the *thermal regime* of detection.

We can represent as in Fig.3-2 the trend of the S/N ratio [standardized to $(2eB)^{1/2}$] as a function of the signal amplitude I_{ph}/I_{ph0}: in the thermal regime the slope is 20 dB/decade up to the threshold $I_{ph}/I_{ph0}=1$, and from here onward the slope becomes 10 dB/decade in the quantum regime. Also shown in Fig.3-2 is the case of an excess noise factor F.

Noise and S/N terminology

The threshold of quantum regime $I_{ph} = I_{ph0} = I_b + (2kT/e)/RFG^2$ is the photodetected signal giving the *break point* between thermal and quantum regimes. When the dark current is very small, the second term is the dominant one; any eventual internal gain greatly helps because it scales the load resistance as G^2.

In all cases, I_{ph0} can be interpreted as the *equivalent dark current* level of the photodetector. Rewriting Eq.(3.11) as:

$$I_{ph0} = (2kT/e) / R_{eq} G^2, \tag{3.11'}$$

with $1/R_{eq} = (1/RF)+G^2 I_b (e/2kT)$, R_{eq} assumes the meaning of noise-equivalent load *resistance* of the photodetector.

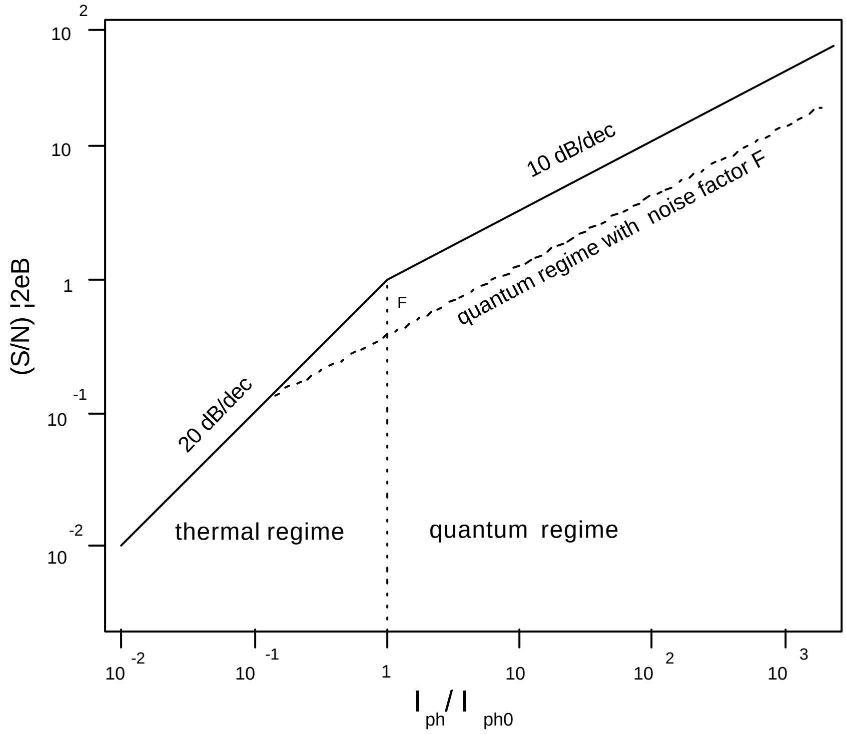

Figure 3-2 The S/N ratio of a quantum photodetector as a function of the input signal in the thermal and quantum regimes of detection

Yet we can define an input noise *equivalent power* as:

$$p_n = (1/\sigma) [2e(I_{ph} + I_b) B F + 4kTB/RG^2]^{1/2} \quad [W] \tag{3.15}$$

and a noise equivalent *power spectral density:*

$$dp_n/df = (1/\sigma) [2e(I_{ph} + I_b) F + 4 kT/RG^2]^{1/2} \quad [W/Hz^{1/2}] \tag{3.15'}$$

Lastly, considering the *noise figure F,* defined as the quotient between the input and output S/N ratios, we have:

$$F = (S/N)_i /(S/N)_u \qquad (3.16)$$

For a quantum photodetector, the noise figure is found from Eqs.(3.10) through (3.12) and can be written as:

$$F^2 = 1+ I_{ph0}/I_{ph} \qquad (3.16')$$

3.3 FIGURES OF MERIT OF DETECTORS

To make a meaningful comparison of the sensitivity performances of quite different detectors (for example: a phototube, the human eye, a photographic film, etc.), Jones [1] introduced in 1952 some figures of merit that soon gained a general acceptance and are widely used to characterize detectors, especially those for the thermal IR.

When considering concept of *sensitivity* of a photodetector, let us point out that it must be kept quite distinct from the concept of response *quantity*. Illustratively, for an amplified phototube that supplies an output voltage V_u in response to the detection of a radiant power P, the ratio V_u/P is not representative of the photodetector sensitivity, simply because it may be increased at will by cascading an amplifier to the phototube.

Such a ratio V_u/P is relevant, however, because it is needed to design the system which uses the output V_u (and therefore P) for a measurement or an actuation, though this quantity is not the inherent sensitivity of the system. To make this distinction clear, the term sensitivity is avoided in describing the detector, and we introduce the *responsivity* R as:

$$R = Q_u / P \qquad (3.17)$$

where Q_u is the output quantity supplied by the photodetector (e.g., a current I_u, a voltage V_u, or any other physical quantity).

3.3.1 NEP and Detectivity

At equal responsivity, the detector with the smallest output noise Q_n on the useful signal is the most sensitive. Therefore, a first figure of merit for a detector is the NEP – or *noise equivalent power* – defined as the ratio of output noise to responsivity:

$$NEP = g_n / R \qquad (3.18)$$

Of course, the NEP represents the input power that gives a unity signal to noise ratio, S/N=1 at the output; that is, a marginal condition of detection.

Since the smaller the NEP is, the better the detector performance is, it is more convenient to define its inverse as a merit figure. This takes the name of *detectivity* D:

$$D = 1/\mathrm{NEP} \tag{3.19}$$

Now, the detectivity D is still dependent on incidental parameters of the detector. Indeed, whatever the noise source is, it can be expected that the noise quadratic total value will be proportional to observation bandwidth B and detector area A.

Example. For a phototube, taking the detected current as the output quantity, the responsivity is $\mathcal{R} = I_u/P = \sigma$, and the noise limit for $P \approx 0$ (i.e. weak signals) is set by the dark current, so that $I_n{}^2 = 2eI_dB$. The dark current I_d can be expressed as $I_d = J_dA$, where J_d is the dark current density (Section 2.6). Thus we have a NEP given by:

$$\mathrm{NEP} = I_n/\mathcal{R} = (1/\sigma)\,(2eJ_dAB)^{1/2} \tag{3.20}$$

with a dependence on AB. In all other detectors, a similar argument applies and the dependence on AB is again found.

Thus, it is even better to take, as the intrinsic noise parameter of a detector, the ratio NEP/ (AB) normalized to unit area and bandwidth. To have a parameter that increases as the performance improves, the detectivity D* (called *D-star*) is defined as:

$$D^* = (AB)/\mathrm{NEP} \tag{3.21}$$

The commonly employed measurement unit for D* is cm $\mathrm{Hz \cdot W^{-1}}$ (although not a SI unit). In Fig.3-3 we report the typical D*s that compare quantum, thermal and other detectors as a function of λ. Note that, given a D* value in Fig.3-3, the output noise from a detector with area A and working on a bandwidth B follows as $Q_n = \mathcal{R}\ (AB)/D^*$.

A final correction to the detectivity comes from the consideration that, in a detector with an angular field of view α, the performance can apparently be better than that of an identical detector with full field of view, $\alpha=\pi/2$. But, a detector with area A and field of view α can always be transformed, by means of an optics, to one with area A' and α'. For the radiance invariance (see Appendix A2), we have $A\Omega = A'\Omega'$ (where $\Omega = \pi \sin^2\alpha = \pi NA^2$). Therefore, it is appropriate to standardize [2] the noise by unit acceptance $A\Omega = NA\ A$ to obtain a figure of merit independent from the detector optics. The D** (*D-double star*) detectivity is accordingly defined as:

$$D^{**} = D^*\,NA = NA\ (AB)/\,\mathrm{NEP} \tag{3.21'}$$

3.3.2 Background Limit or BLIP

In the infrared, the photodetector is usually aimed toward a scene at a temperature T, which we may take first as a blackbody (ε=1) with a radiance $r(\lambda)=dR/d\lambda$ per unit wavelength and unit area as given by Planck law [see Eq.(A1.6)]:

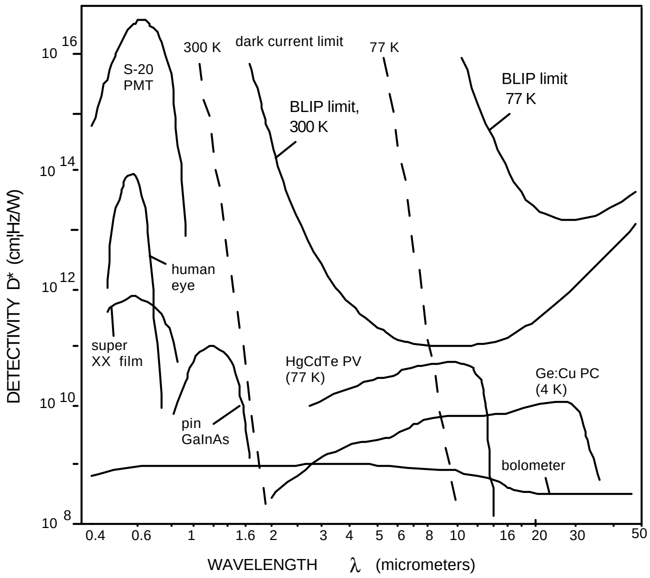

Figure 3-3 Detectivity as a function of wavelength for a number of different photodetectors. The BLIP and the dark current limits are indicated.

$$r(\lambda) = 2h\nu^2/\lambda^3(\exp h\nu/kT - 1) \qquad (3.22)$$

From Eq.(3.22), applying the law of photography (Appendix A2), the detected current density follows as:

$$J_{bg} = \sigma\, r(\lambda)\, \Delta\lambda\, \pi NA^2$$

This current is itself a dc component, easily filtered out, but also carries a shot noise given by:

$$i^2_{n\,(bg)} = 2\, e\, J_{bg}\, A\, B \qquad (3.23)$$

Since NEP=$i_{n(bg)}/\sigma$, by inserting Eq.(3.23) in Eq.(3.21') we get a limit in detectivity D** due to the background shot noise, for this reason called the BLIP limit (*background-limited-intrinsic-performance*), given by:

$$D_{BLIP} = [\sigma / 2\pi e\, r(\lambda)\Delta\lambda] \tag{3.24}$$

The behaviour of D_{BLIP} for T=300 K and 77 K at $\Delta\lambda$=1µm is reported in Fig.3-3, and represents the best sensitivity that can be obtained from an infrared photodetector when it works looking at the thermal background. At ambient temperature T=300 K, the minimum D_{BLIP} is around $\lambda \approx$10µm, and its behaviour versus λ is basically that of the black body drawn upside down.

The D_{BLIP} does not actually represent an absolute limit for a photodetector, but even in the case D**>D_{BLIP}, the photodetector aimed at a scene at temperature T would have a noise performance determined by the background, i.e. by D_{BLIP}. For this reason, in IR detectors intended for thermography, is it useless to get D* much better than D_{BLIP}, as indeed shown by the typical performances of Fig.3-3 (more data are given in Section 6.2).

On the other hand, the BLIP is of little significance for visible and NIR detectors, where the background level becomes negligibly small (or, the D_{BLIP} is very high). In this case the limit of performance is better described by the arguments of Section 3.2. In particular, if the Johnson noise is negligible, it is the dark current that dominates the performance.

By inserting in Eq.(3.20) the dark current density [Eq.(2.16')] for a photoemissive detector with threshold λ_t, J_d=H exp-hc/λ_tkT, and using the definition [Eq.(3.21)] we have for the detectivity of a quantum detector with photoelectric threshold λ_t limited by the dark current:

$$D_{dark} = \sigma(2eH)^{-1/2} \exp\,(hc/2\lambda_t\, kT) \tag{3.25}$$

The resulting D_{dark} is shown in Fig.3-3 where we can see that, because of the very steep dependence on λ, the noise performance changes from background to dark current dominated, below a certain wavelength dictated by temperature.

References

[1] R.C. Jones: "*Performances of Detectors for Visible and Infrared Radiation*", in: Advances in Electronics and Electron Physics, vol.4, pp.2-88, Academic Press, New York, 1952.

[2] R.D. Hudson: "*Infrared System Engineering*", Wiley-Interscience, New York, 1969.

[3] B.M. Oliver: "*Thermal and Quantum Noise*", in: Electrical Noise, Fundamentals and Sources, edited by M.S. Gupta, IEEE Selected Reprint Series, New York, 1977.